ANALYSIS OF ALGEBRA PLACEMENT AT CALIFORNIA STATE UNIVERSITY LONG BEACH

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Kirsten Miller

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I, THE UNDERSIGNED MEMBER OF THE COMMITTEE,

HAVE APPROVED THIS THESIS

ANALYSIS OF ALGEBRA PLACEMENT AT CALIFORNIA STATE UNIVERSITY LONG BEACH

BY

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Table of Contents

Abstract

In an attempt to increase four-year graduation rates, beginning Fall of 2018, the California State University system will eliminate system-wide placement exams and remedial courses for freshmen. This means that the twenty-three campuses in the California State University system will need to find their own placement mechanism for placing freshmen in algebra-intensive mathematics courses such as Precalculus and Calculus. In 2016, the Long Beach campus introduced the ALEKS (Assessment and Learning in Knowledge Spaces) program to accomplish this task. This study answers the following questions: What cut scores for the ALEKS mathematics assessment and STEM Eligibility Index would result in successful placement of students in pre-technical (science, technology, engineering, mathematics, and business) majors in Precalculus Algebra (Math 113)? Does the current ALEKS cut score of 46 for Math 113 produce the expected pass rate for that course? What impacts do high school GPA, performance on ALEKS assessments, and STEM Eligibility Index score have on a student's probability of passing Math 113? A data set was obtained containing ALEKS scores, SAT scores, high school GPA, and final grades in Precalculus Algebra (Math 113) for pre-technical students in the freshman class of 2017. A conditional probability analysis revealed that given a student meets or exceeds the cut score of 46 on the ALEKS assessment, he or she will pass Math 113 with a probability of 0.7528. Conditional probability analysis of ALEKS scores and STEM Eligibility Index scores as well as failure rates over STEM Eligibility Index score ranges led to the recommendation that students with an ALEKS score between 46 and 50 should have a STEM Eligibility Index score of 3500 rather than the minimum 3300 in order to enroll in Math 113. The results of the logistic regression revealed that STEM Eligibility Index is a better predictor of

success than high school GPA; therefore, STEM Eligibility Index should be used to make placement decisions for students with borderline ALEKS scores, scores in the 46-50 range.

Literature Review

Collegiate Mathematics Placement

Placement of Freshmen into their first collegiate mathematics course is an important decision with significant ramifications. Universities strive to find a placement strategy that avoids both overplacement and underplacement of students (Woods, 2017). Overplacement refers to placing a student into a course for which they have not yet developed the background knowledge and mathematical skill set necessary to be successful, making failure likely. This can have serious implications in terms of retention rates, for a study conducted on engineering students at Boise State University found a strong correlation between students' grades in their first collegiate mathematics course and their likelihood of continuing in an engineering major. The level of mathematics at which they started however did not affect their likelihood of remaining in an engineering program (Gardner, Pyke, Belcheir, & Schrader, 2007). Underplacement refers to placing a student into a mathematics course for which he or she is overqualified and therefore will derive little academic growth. This comes with the danger of extending the timeline necessary for mathematics prerequisites to be fulfilled, increasing time required to complete a degree, and increasing overall costs of tuition and course materials. Thus with these potential hazards in mind, colleges and universities search for a mechanism that will place students in appropriate mathematics courses on their campus.

In 2011, the National Assessment Governing Board (NAGB) **—** "an independent body of state and local educators, policymakers, technical experts, business leaders, parents, and the general public" (Fields & Parsad, 2012) — conducted a survey involving 1,560 public and private post-secondary institutions in order to determine which standardized tests and cut scores were being used to determine whether entering students were in need of remediation. The study revealed that 85% of four-year public institutions used a standardized mathematics exam for this purpose: 31% used the ACT and 22% used the SAT. From the reports of all 1,560 participating schools, it was found that mean cut scores used were 19 on the mathematics portion of the ACT (scale: 1-36) and 471 on the mathematics section of the old SAT (scale: 200-800).

Mathematics Placement at CSULB

Historically, California State University Long Beach (CSULB) placed incoming students into mathematics classes via multiple paths which include the following: earning a score on the California Assessment of Student Performance and Progress (CASPP) Early Assessment Program (EAP) exam that categorizes them as “Ready” for California State University (CSU) coursework, earning a score on the CASPP EAP exam that categorizes them as “Conditionally Ready” for CSU coursework and earning a grade of “C” or better in an approved math course in their 12th grade year, scoring above 520 on the new SAT Test, scoring above 490 on the old SAT Reasoning Test, scoring above 20 on the mathematics portion of the ACT Test, scoring a 3 or greater on the College Board Advanced Placement Calculus AB, Calculus BC, or Statistics exam, or earning a grade of “C” or better in a transferable college Quantitative Reasoning course (CSULB, 2017). Students who failed to demonstrate preparation for college level mathematics by satisfying at least one of the above criterium were required to take the Entry Level Mathematics (ELM) Exam, a traditional paper and pencil exam that tested students for proficiency in Algebra I, Algebra II, and Geometry. Obtaining a score of 50 or greater on this exam placed students into college-level mathematics courses. Scoring below the cut score of 50 resulted in placement in remedial mathematics course(s).

In 2016, CSULB started using the ALEKS PPL (Placement, Preparation, and Learning) product for Calculus I placement. Incoming freshmen who were classified as “GE Math Ready”

— scored 50 or greater on the ELM, 550 or above on the old SAT, 570 or above on the new SAT, 23 or above on the ACT, or a score on the EAP that classified them as “Ready” (CSULB, 2017) — and declared a pre-major that requires Calculus I or Survey of Calculus were given free access to the ALEKS PPL product. Those students were expected to complete four ALEKS assessments and spend time in the adaptive learning module between each assessment in order to review topics and build mathematical skills that were underdeveloped. Topics covered on the ALEKS Calculus Readiness Exam included: Rational Expressions and their graphs, Radicals, Exponents, Functions, Polynomials, Geometric Applications, Linear Equations, Linear Inequalities, Absolute Values, Logarithms, Exponential Equations, and Trigonometry (CSULB, n.d.).

CSULB expanded their use of the ALEKS program in 2017 when they implemented it in their Summer Early Start Math program. Students who failed to achieve the classification of “GE Math Ready” were required to participate in a CSU Early Start Program. To fulfill this requirement, the Long Beach campus offered a one-unit course in which students met for three hours and forty-five minutes, once a week, for a four-week period and worked in the learning module of the ALEKS program in order to strengthen foundational mathematical skills (CSULB, n.d.). Students were strongly encouraged to spend time on self-remediation between class meetings. During the course of the four-week session, students took up to five placement exams in the ALEKS program in hopes of testing out of remedial math by achieving a score of 46 or greater.

ALEKS PPL and Implementation

ALEKS (Assessment and Learning in Knowledge Spaces) “is a Web-based, artificially intelligent assessment and learning system [which] uses adaptive questioning...” (ALEKS, 2018).

This program was designed on the basis of knowledge space theory which asserts that a knowledge state — a detailed outline of what a student knows and has yet to learn — can be used to represent a student's proficiency in a particular subject at any point in time. The set of every possible knowledge state is referred to as a knowledge space (Woods, 2017). When the ALEKS program was designed, the assessments included in this product were not intended for use as placement examinations. The initial assessment was required to determine each individual user's knowledge state and set up an appropriate learning module for that student. Subsequent assessments identified the student's new knowledge state as he or she progressed and kept the program appropriately calibrated for the user (Ahlgren & Harper, 2011). This adaptive characteristic of ALEKS makes the system extremely efficient and informative, for with just 30 questions, ALEKS can measure a student's level of proficiency in knowledge domains that encompass 200-300 mathematical items (Harper & Reddy, 2013). Examples of mathematical items include plotting exponential functions and solving quadratic equations (Ahlgren & Harper, 2011).

Motivated by high failure rates in Calculus I, in 2007, the University of Illinois changed their placement method from use of ACT scores to ALEKS scores. Factors contributing to this decision included (1) data from Louisiana Tech University which found a correlation between higher ALEKS assessment scores and higher course grades and (2) the ability of ALEKS to provide students with an opportunity for self-remediation before classes began (Ahlgren & Harper, 2011). Achieving the required ALEKS score was the only prerequisite for five mathematics courses on this campus: College Algebra, PreCalculus, Business Calculus, Calculus I, and Calculus I for experienced students (Harper & Reddy, 2013). Grades in previous courses were not taken into account for enrollment decisions, only an ALEKS score obtained within four months of the start date of the desired course (Ahlgren & Harper, 2011). This policy was intended to ensure that students were placed in courses based on their current mathematical knowledge and level of understanding, not the knowledge they possessed at their peak and may have lost.

The University of Illinois analyzed data collected over the course of three years (this included over 20,000 ALEKS assessments) and found "very high correlations between mean grades over small ALEKS score ranges and range midpoints" (Ahlgren & Harper, 2011). This result supported their initial assertion that ALEKS effectively determines a student's knowledge state before he or she enters a course, and a student's level of knowledge should be a good predictor of success in a course (Ahlgren & Harper, 2011). With continued use of ALEKS assessments for placement decisions, the university observed a decrease in failure and withdrawal rates. Because of this success, other universities began adopting ALEKS-based placement procedures. These schools include University of Arizona, Arizona State, University of Colorado, University of Florida, University of Missouri, Michigan Technological University, University of Texas, and Perdue University.

Using knowledge space theory for placement requires the convenience associated with a numerical score. Thus the ALEKS system uses the following method to produce such a score: ALEKS takes the cardinality of the student's knowledge state and divides that by the total number of mathematical items in the domain which produces a percentage representing the portion of the domain that the student has mastered (Harper & Reddy, 2013). Universities utilizing ALEKS-based placement then compare that score to their established cut scores. The University of Illinois initially used a cut score of 40 for their Precalculus and Business Calculus courses and a cut score of 60 for Calculus I placement; however, data indicated that students with ALEKS scores above 50 and 70, respectively, were more likely to be successful in those courses. Thus they made appropriate adjustments to their cut scores (Woods, 2017). The current cut score for Precalculus Algebra at CSULB is 46.

STEM Eligibility Index

Applicants for STEM (Science, Technology, Engineering, and Math) majors at CSULB are required to obtain a STEM Eligibility Index score of 3300 or greater. This score is calculated using the following formula:

(College Preparatory Grade Point Average x 600) + SAT Critical Reading + (2 x SAT Math)

Scores from the old SAT are used in this formula. Thus new SAT scores and ACT scores are converted to their old SAT score equivalents using concordance tables published by the College Board. College Preparatory Grade Point Average (GPA) is calculated using grades obtained in CSU approved college preparatory courses taken in grades 10-12. Additional weight is added to a maximum of eight approved honors courses taken after ninth grade, including a maximum of two honors, Advanced Placement (AP), or International Baccalaureate (IB) courses taken in tenth grade (CSULB, n.d.). This formula puts the largest weight on GPA, considering it an indicator of study skills. Mathematics scores are also given considerable weight because STEM majors require developed quantitative reasoning skills (CSULB, n.d.).

Methodology

A set of de-identified data for the freshman class of 2017 was obtained from the CSULB Mathematics Department Faculty. The data set included students' highest ALEKS assessment score, high school GPA, SAT verbal score, SAT math score, ACT verbal score, ACT math score and final grade in Math 113 (Precalculus Algebra). The sample size for this study was 321 Math 113 students who had recorded ALEKS scores and SAT or ACT scores.

Data Preparation and Coding

Because the STEM Eligibility Index is calculated using old SAT scores, new SAT scores and ACT scores were converted to old SAT scores using concordance tables published by Utah State University. The analysis techniques used in the study require binary outcomes; therefore, final grades for Math 113 were placed into one of two categories: pass or fail. Grades of A, B, C, or CR were categorized as passing grades and labeled with a 1, the conventional notation for a successful outcome in a statistical experiment. Grades of D, F, NC, W, WU, or WE were categorized as failing grades and labeled with a 0, the conventional notation for failure in a statistical experiment.

ALEKS Cut Scores and Pass Rates

One of the questions of this study was: Does the current ALEKS cut score of 46 for Math 113 produce the expected pass rate for that course? Two forms of analysis were applied to the data to answer this question, the first of which was conditional probability analysis. This test was constructed to determine the probability of receiving a passing (A, B, C, or CR) or failing grade (D, F, NC, W, WU, or WE) in Math 113 given an ALEKS score above or below a particular pivot. The second form of analysis determined failure rates among Math 113 students in various ALEKS score ranges. Below is a more thorough explanation of each analysis technique.

Conditional Probability Analysis of ALEKS Scores

Relevant data for this analysis included students' highest ALEKS placement exam scores and students' final grades in Math 113, coded as passing (1) or failing (0). Using the conditional probability formula, two probabilities were calculated:

1) The probability of a student receiving a passing grade in Math 113, given that his or her highest ALEKS placement score was above a particular pivot score

$$P\,(\text{Passing Grade} \mid \text{Score} \geq \text{Pivot}) = \frac{P\,(\text{Passing Grade and Score} \geq \text{Pivot})}{P\,(\text{Score} \geq \text{Pivot})}$$

Where P (Score $\geq$ Pivot) is the probability that a randomly selected student in the data set scored an ALEKS score equal to or above a chosen pivot score.

2) The probability of a student receiving a failing grade in Math 113, given that his or her highest ALEKS placement score was below a particular pivot score

$$P\,(\text{Failing Grade} \mid \text{Score} < \text{Pivot}) = \frac{P\,(\text{Failing Grade and Score} < \text{Pivot})}{P\,(\text{Score} < \text{Pivot})}$$

Where P (Score < Pivot) is the probability that the highest ALEKS score of a randomly selected student in the data set is below a chosen pivot score.

This analysis was performed sixteen times with sixteen different pivot values (even numbers between forty and seventy, inclusive). This analysis was performed to verify the expectation that the probability of a student passing Math 113 given that their ALEKS score was above a pivot would increase as the pivot increased (i.e., the higher a student's score on the placement exam, the more likely he or she is to pass the course), and that the probability of a student failing Math 113 given that their ALEKS score was below a pivot would decrease as the pivot increased (i.e., the higher a student's score on the placement exam, the less likely he or she is to fail the course). Upon verification of this expectation, corresponding probabilities at the pivot cut scores could be examined to determine whether they were too high or too low, and appropriate adjustments could be made to the Math 113 cut score.

Failure Rates in ALEKS Score Ranges

The second form of analysis applied to answer the question “Does the current ALEKS cut score of 46 for Math 113 produce the expected pass rate for that course?” was a calculation of Math 113 failure rates among various ALEKS scores ranges. Relevant data for this analysis included students’ highest ALEKS placement exam scores and students’ final grades in Math 113, coded as passing (1) or failing (0). The score ranges were as follows: 0-46 exclusive, 46-48, 49-51, 52-54, 55-57, 58-60, 61-63, 64-66, 67-69, and 70 -100 (note: the 2nd - 10th ranges were inclusive). Failure rates were calculated according to the following formula:

$$\text{Failure Rate} = \frac{\text{Number of Students with an ALEKS score in a certain range who failed Math 113}}{\text{Number of Students with an ALEKS score in a certain range}}$$

The expectation is that failure rates will decrease as ALEKS score ranges increase. Upon verification of this expectation, failure rates can be examined over each ALEKS score range. If they are found to be particularly high in a specific score range or combination of score ranges, appropriate adjustments to the Math 113 cut score can be made.

STEM Eligibility Index and Pass Rates

Another question of this study was: “What cut scores for the ALEKS mathematics assessment and STEM Eligibility Index would result in successful placement of students in pre-technical majors in Math 113?” The two forms of analysis conducted on ALEKS assessment scores above provided useful information to address that portion of the question. Thus similar analysis was conducted on STEM Eligibility Index scores to address the second portion of this question.

Conditional Probability Analysis of STEM Eligibility Index

Relevant data for this analysis included students' high school GPAs, highest old SAT scores, and final grades in Math 113, coded as passing (1) or failing (0). STEM Eligibility Index Scores were calculated using the formula:

(College Preparatory Grade Point Average x 600) + SAT Critical Reading + (2 x SAT Math)

Using the conditional probability formula, two probabilities were calculated:

1) The probability of a student receiving a passing grade in Math 113, given that his or her STEM Eligibility Index score was above a particular pivot score

   $$P\,(\text{Passing Grade} \mid \text{Score} \geq \text{Pivot}) = \frac{P\,(\text{Passing Grade and Score} \geq \text{Pivot})}{P\,(\text{Score} \geq \text{Pivot})}$$

   Where P (Score $\geq$ Pivot) is the probability that a randomly selected student in the data set scored a STEM Eligibility Index score equal to or above a chosen pivot score.

2) The probability of a student receiving a failing grade in Math 113, given that his or her STEM Eligibility Index score was below a particular pivot score

   $$P\,(\text{Failing Grade} \mid \text{Score} < \text{Pivot}) = \frac{P\,(\text{Failing Grade and Score} < \text{Pivot})}{P\,(\text{Score} < \text{Pivot})}$$

   Where P (Score $<$ Pivot) is the probability that the STEM Eligibility Index score of a randomly selected student in the data set is below a chosen pivot score.

This analysis was performed twelve times with twelve different pivot values (numbers between 3100 and 4200, inclusive, incremented by 100). This analysis was performed to verify the expectation that the probability of a student passing Math 113 given that their STEM Eligibility Index score was above a pivot would increase as the pivot increased (i.e., the higher a student's STEM Eligibility Index score, the more likely he or she is to pass the course), and that the probability of a student failing Math 113 given that their STEM Eligibility Index score was below a pivot would decrease as the pivot increased (i.e., the higher a student's STEM Eligibility Index score, the less likely he or she is to fail the course). Upon verification of this expectation, corresponding probabilities at the pivot cut scores could be examined to determine whether they were too high or too low, and useful insights could be provided.

Failure Rates in STEM Eligibility Index Score Ranges

Further insight regarding STEM Eligibility Index scores was provided through the calculation of Math 113 failure rates over STEM Eligibility Index Score Ranges. Relevant data for this analysis included students' high school GPAs, highest old SAT scores, and final grades in Math 113, coded as passing (1) or failing (0). The score ranges were as follows: below 3300, 3300-3400, 3400-3500, 3500-3600, 3600-3700, 3700-3800, 3800-3900, 3900-4000, 4000-4100, and greater than or equal to 4100 (note: the 2nd - 9th ranges were inclusive on the left endpoint and exclusive on the right endpoint). Failure rates were calculated according to the following formula:

$$\text{Failure Rate} = \frac{\text{Number of Students with a STEM EI score in a certain range who failed Math 113}}{\text{Number of Students with a STEM EI score in a certain range}}$$

The expectation is that failure rates will decrease as STEM Eligibility Index score ranges increase. Upon verification of this expectation, failure rates can be examined over each STEM Eligibility Index score range. If they are found to be particularly high in a specific score range or combination of score ranges, suggestions can be made about the implementation of a STEM Eligibility Index score requirement for Math 113.

Logistic Regression

The third question of this study was: "What impacts do high school GPA, performance on ALEKS assessments, and STEM Eligibility Index score have on a student's probability of passing Math 113?" In order to address this question, a logistic regression model was used. The model took the form of:

Logit [P(Y)] = $\alpha + \beta_1$ (GPA) + β_2(ALEKS Score) + β_3(STEM Eligibility Index Score)

Where :

Logit [P(Y)] = ln $[\frac{P(X=1)}{1 - P(X=1)}]$

P(X = 1) is the probability that a student will pass Math 113

α is a constant that provides the log of the baseline odds. This is the value for logit [P(Y)] that would be achieved if a student had a GPA, ALEKS Score, and STEM Eligibility Index Score of 0.

β_1, β_2, and β_3 are constants that provide the overall change in logit [P(X=1)] that would result from a one unit change in GPA, ALEKS score, and STEM Eligibility Index Score, respectively

This form of analysis was selected because logistic regression is a mathematical model that is used to describe the relationship between multiple independent inputs and one dependent, dichotomous output (Kleinbaum & Klein, 2010), in the case of this study, passing or failing Math 113.

The coefficients α, β_1, β_2, and β_3 were found using the MATLAB (Matrix Laboratory) built-in function, "mnrfit." Relevant data for this study included a set of three inputs, students' high school GPAs, ALEKS placement exam scores, and STEM Eligibility index scores, and one output, students' corresponding grades in Math 113, coded as pass (1) or fail (0).

Results

Conditional Probability Analysis of ALEKS Scores

Table 1 contains the computed values for the probability of a student receiving a passing grade in Math 113, given that his or her highest ALEKS placement score was greater than or equal to a particular pivot score. In this particular analysis, the expectation was that

P (Student Earned a Passing Grade in Math 113 |Student's ALEKS Score $\geq$ Pivot Score) would increase as the pivot score increased. Overall, the data supports this expectation with a few minor exceptions at 48-50 and 52-54. Based on the data, students who attain or exceed the ALEKS cut score of 46 should pass Math 113 with a probability of at least 0.7528.

Table 2 contains the computed values for the probability of a student receiving a failing grade in Math 113, given that his or her highest ALEKS placement score was below a particular pivot score. The expectation for this form of analysis was that

P (Student Received a Failing Grade in Math 113 | Student's ALEKS Score $<$ Pivot Score) would decrease as the pivot score increased. Overall, the data supports this expectation with exceptions at 40-42, 50-52, and 58-60. Based on the data, students placed in Math 113 with ALEKS scores below the cut score of 46 will fail the class with a probability of at least 0.5926.

Pivot Score	P (Passing Grade | Score $\geq$ Pivot)	Number of Students who Scored At or Above the Pivot
40	0.7352	287
42	0.7456	283
44	0.7509	277
46	0.7528	267
48	0.7589	253
50	0.7562	242
52	0.7783	230
54	0.7763	219
56	0.7854	205
58	0.7850	200
60	0.7969	192
62	0.8079	177
64	0.8121	165
66	0.8182	143
68	0.8321	131
70	0.8512	121

Table 1. *Results of a conditional probability analysis of ALEKS Assessment scores. The analysis was performed on data from 321 freshmen Math 113 students with recorded ALEKS and SAT/ACT scores. A MATLAB program was written for this study and used to calculate the probability that a student will pass Math 113 given that his or her highest ALEKS Assessment score is above a particular pivot. The pivot scores studied were even numbers between 40 and 70, inclusive. The expectation was that the conditional probability would increase as the pivot scores increased.*

Figures 1 and 2 are graphical representations of the information presented in Table 1.

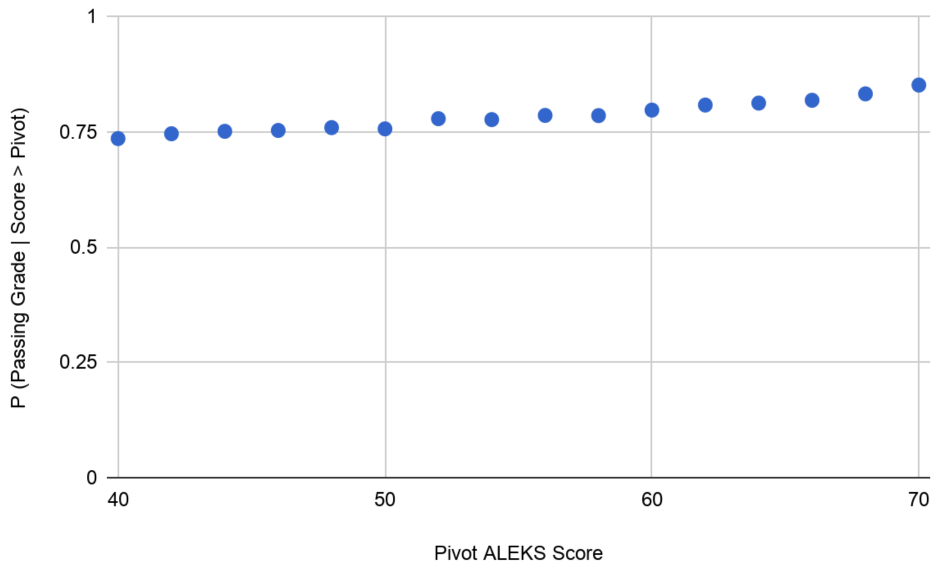

Figure 1. *Graphical representation of results of conditional probability analysis of ALEKS Assessment scores. Conditional probability of success in Math 113 (vertical axis) generally increases as pivot ALEKS scores (horizontal axis) increase, confirming the expectation.*

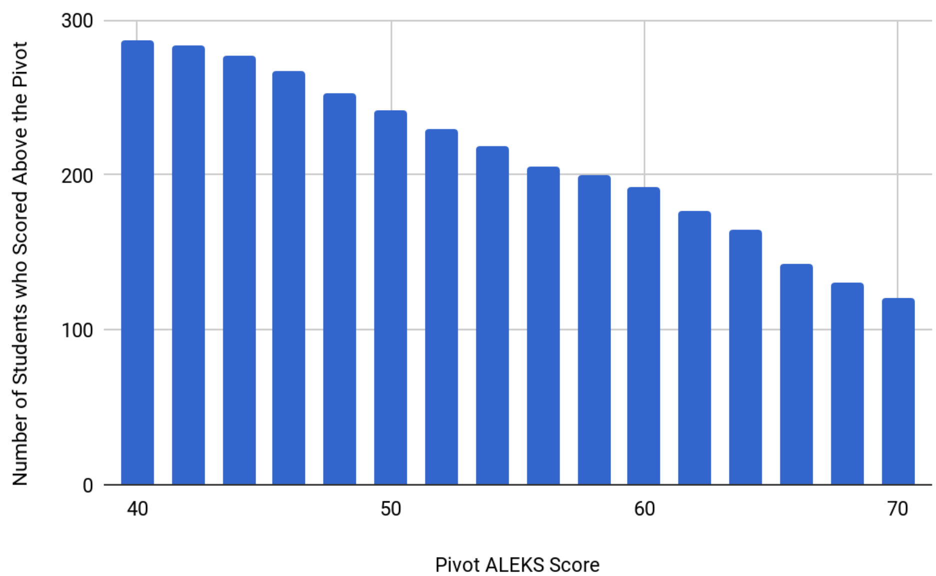

Figure 2. *Number of freshmen Math 113 students (vertical axis) whose highest ALEKS Assessment score was greater than or equal to each pivot score studied (horizontal axis) — even numbers between 40 and 70, inclusive.*

Pivot Score	P (Failing Grade | Score < Pivot)	Number of Students who Scored Below the Pivot
40	0.6471	34
42	0.6842	38
44	0.6591	44
46	0.5926	54
48	0.5441	68
50	0.4937	79
52	0.5165	91
54	0.4804	102
56	0.4655	116
58	0.4545	121
60	0.4574	129
62	0.4444	144
64	0.4295	156
66	0.4045	178
68	0.4000	190
70	0.4000	200

Table 2. *Results of a conditional probability analysis of ALEKS Assessment scores. A MATLAB program, written for this study, was used to calculate the probability that a student would fail Math 113 given that his or her highest ALEKS Assessment score was below a particular pivot. The pivots studied were even numbers between 40 and 70, inclusive. The expectation was that conditional probability would decrease as pivot scores increased. Data for this analysis came from 321 freshmen Math 113 students with recorded ALEKS scores and SAT/ACT scores.*

Figures 3 and 4 are graphical representations of the information presented in Table 2.

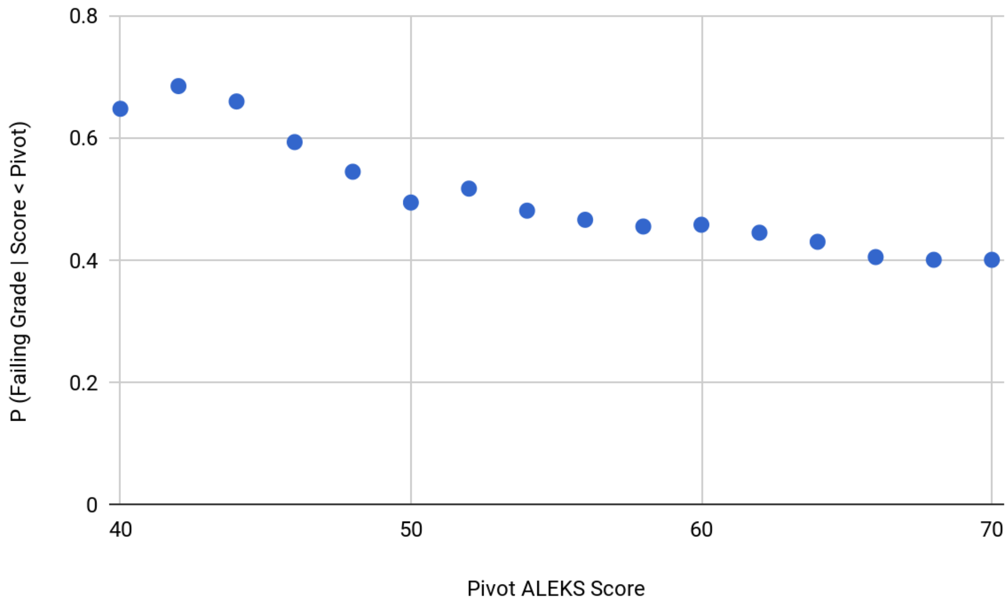

Figure 3. *Graphical representation of the results of the conditional probability analysis of ALEKS Assessment scores. The conditional probability of failing Math 113 (vertical axis) generally decreases as the pivot ALEKS score (horizontal axis) increases, confirming the expectation.*

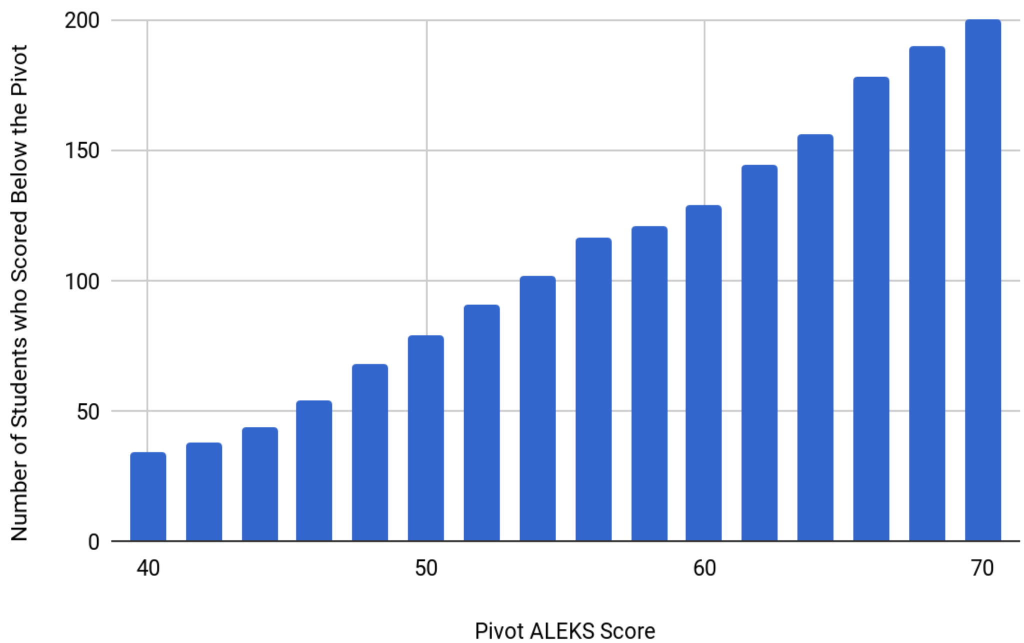

Figure 4 *Number of freshmen Math 113 students (vertical axis) with ALEKS Assessment scores below the pivots studied (horizontal axis) — even numbers between 40 and 70, inclusive.*

Failure Rates in ALEKS Score Ranges

Table 3 includes computed Math 113 failure rates over ten ALEKS score ranges. The expectation was that failure rates would decrease as the ALEKS scores increased; however, this trend was not observed (see Figure 3.1 for a visual demonstration). This failure to produce a clear trend is due to small sample sizes in a majority of score ranges. Sample sizes were less than or equal to 20 in the following ALEKS score ranges: 46-48, 49-51, 52-54, 55-57, 58-60, 61-63, and 67-69, thus results in these ranges are fairly unreliable. The data however reliably demonstrates that students with an ALEKS score below the cut score of 46 are likely to fail Math 113 with probability 0.5926 and students with ALEKS scores greater than or equal to 70 were unlikely to fail Math 113 with probability 0.1488.

ALEKS Score Range	Failure Rate	Failure Rate Expressed as a Percentage	Number of Students with ALEKS Scores in the Range
0 - 45	0.5926	59.26	54
46 - 48	0.3158	31.58	19
49 - 51	0.5000	50.00	18
52 - 54	0.1765	17.65	17
55 - 57	0.3846	38.46	13
58 - 60	0.5333	53.33	15
61 - 63	0.2000	20.00	20
64 - 66	0.2143	21.43	28
67 - 69	0.4375	43.75	16
70 - 100	0.1488	14.88	121

Table 3. *Results of failure rate analysis in ALEKS Assessment score ranges. Data from the 321 freshmen Math 113 students used in the study was divided into 10 categories based on students' highest ALEKS Assessment scores. Within the ALEKS score ranges, Math 113 failure rates were calculated. The expectation was that failure rates would decrease as ALEKS scores increased.*

Figures 5 and 6 are graphical representations of the information presented in Table 3.

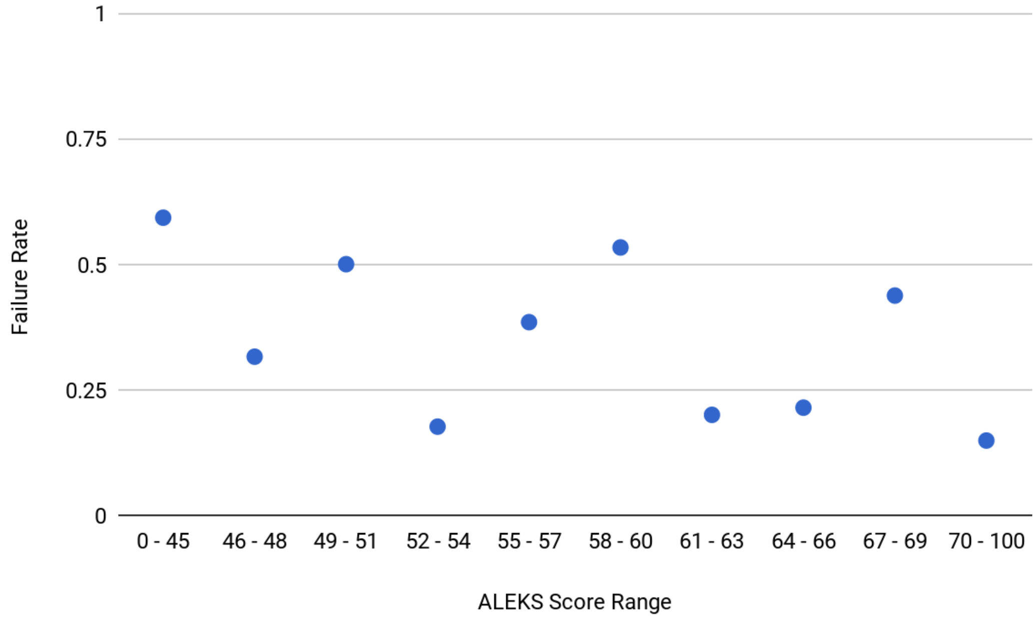

Figure 5. *Graphical representation of results of failure rate analysis in ALEKS score ranges. A clear trend is not displayed thus the expectation of failure rates (vertical axis) decreasing as ALEKS scores (horizontal axis) increased could not be confirmed, likely due to small sample sizes in ALEKS score ranges.*

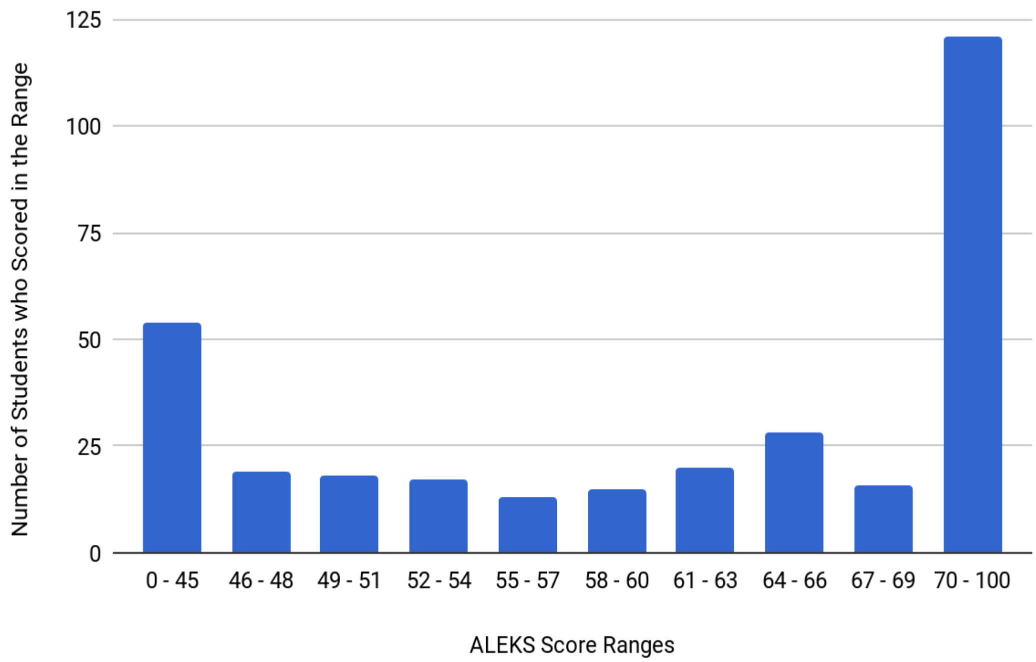

Figure 6. *Number of students with ALEKS Assessment scores in each score range. Graphically demonstrates the small sample sizes in ALEKS score ranges between 46 and 69.*

Conditional Probability Analysis of STEM Eligibility Index

Table 4 contains the computed values for the probability of a student receiving a passing grade in Math 113, given that his or her STEM Eligibility Index score was greater than or equal to a particular pivot score. The expectation for this analysis was that P (Student Earned a Passing Grade in Math 113 | Student's STEM EI Score $\geq$ Pivot Score) would increase as the pivot score increased. The data demonstrated this trend without exception. Based on the data, students who achieve or exceed CSULB's minimum STEM Eligibility Index score for STEM majors, 3300, will pass Math 113 with a probability of at least 0.7105. A noteworthy observation is that the data indicates that students with a STEM Eligibility Index score greater than or equal to 4200 will pass Math 113 with a probability of 1. A probability of 1 indicates certainty; however, it is not true that a student with a STEM Eligibility Index score greater than or equal to 4200 is guaranteed to pass Math 113. This result stems from a small sample size of 12 students scoring at or above 4200.

Table 5 contains the computed values for the probability of a student receiving a failing grade in Math 113, given that his or her STEM Eligibility Index score was less than a particular pivot score. The expectation for this analysis was that P (Student Receives a Failing Grade in Math 113 | Student's STEM EI Score $<$ Pivot Score) would decrease as the pivot score increased. Overall, this expectation was supported by the data with an exception at 3100-3200. Based on the data, students who do not achieve CSULB's minimum STEM Eligibility Index score for STEM majors, 3300, will fail Math 113 with a probability of at least 0.5000.

Pivot Score	P (Passing Grade | Score $\geq$ Pivot)	Number of Students who Scored At or Above the Pivot
3100	0.6959	319
3200	0.6994	316
3300	0.7105	304
3400	0.7298	285
3500	0.7668	253
3600	0.8086	209
3700	0.8788	165
3800	0.9030	134
3900	0.9293	99
4000	0.9394	66
4100	0.9688	32
4200	1	12

Table 4. *Results of a conditional probability analysis of STEM Eligibility Index scores. The analysis was performed on data from 321 freshmen Math 113 students with recorded ALEKS and SAT/ACT scores. A MATLAB program was written for this study and used to calculate the probability that a student will pass Math 113 given that his or her STEM Eligiblity Index score is above a particular pivot. The pivot scores studied were between 3100 and 4200, inclusive, incremented by 100. The expectation was that the conditional probability would increase as the pivot scores increased.*

Figures 7 and 8 are graphical representations of the information presented in Table 4.

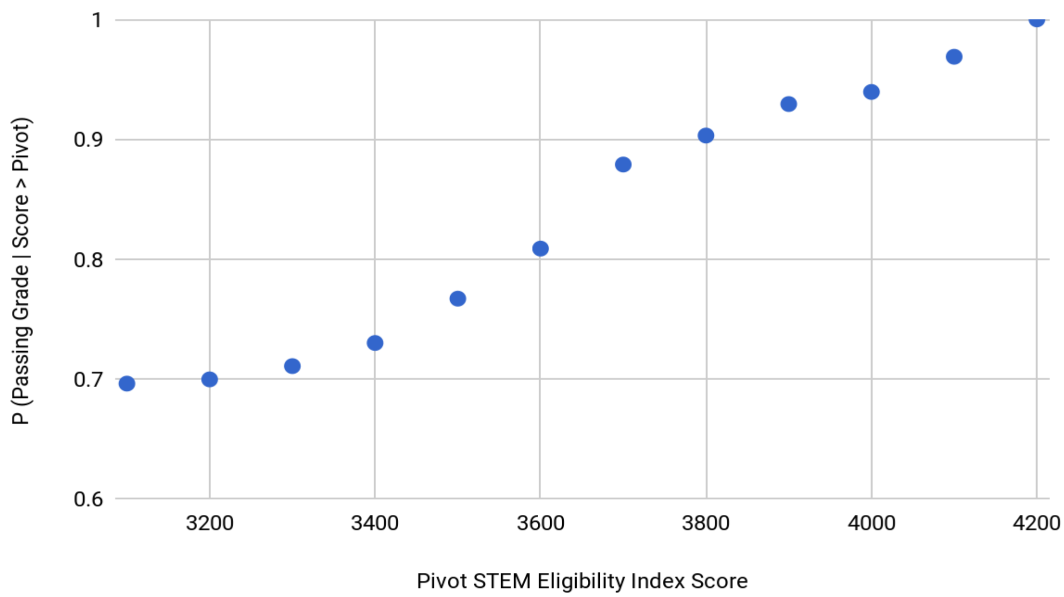

Figure 7. *Graphical representation of the results of a conditional probability analysis of STEM Eligibility Index scores. The conditional probability of a student passing Math 113 (vertical axis) generally increases as the STEM EI score (horizontal axis) increases, confirming the expectation.*

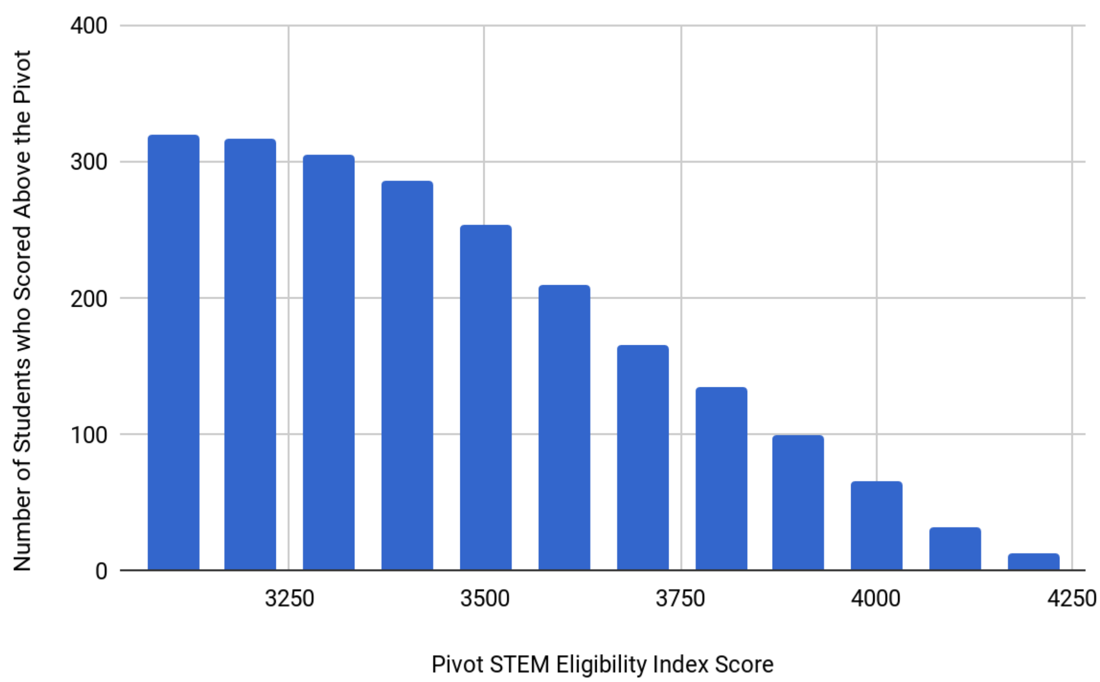

Figure 8. *Number of students (vertical axis) with STEM Eligibility Index scores (horizontal axis) at or above each pivot — numbers between 3100 and 4200, inclusive, incremented by 100.*

Pivot Score	P (Failing Grade | Score < Pivot)	Number of Students who Scored Below the Pivot
3100	0.5000	2
3200	0.6000	5
3300	0.5882	17
3400	0.5833	36
3500	0.5735	68
3600	0.5179	112
3700	0.5000	156
3800	0.4545	187
3900	0.4099	222
4000	0.3686	255
4100	0.3356	289
4200	0.3172	309

Table 5. *Results of a conditional probability analysis of STEM Eligibility Index scores. A MATLAB program, written for this study, was used to calculate the probability that a student would fail Math 113 given that his or her STEM Eligibility Index score was below a particular pivot. The pivots studied were between 3100 and 4200, inclusive, incremented by 100. The expectation was that conditional probability would decrease as pivot scores increased. Data for this analysis came from 321 freshmen Math 113 students with recorded ALEKS scores and SAT/ACT scores.*

Figures 9 and 10 are graphical representations of the information presented in Table 5.

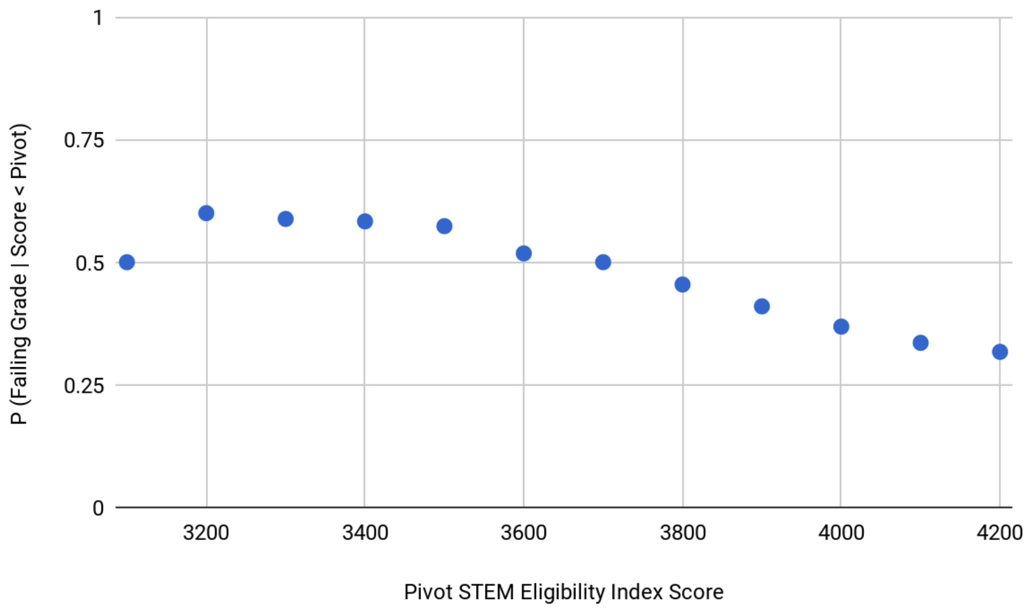

Figure 9 . *Graphical representation of the results of the conditional probability analysis of STEM Eligibility Index scores. The conditional probability of failing Math 113 (vertical axis) generally decreases as pivot STEM EI scores (horizontal axis) increase, confirming the expectation.*

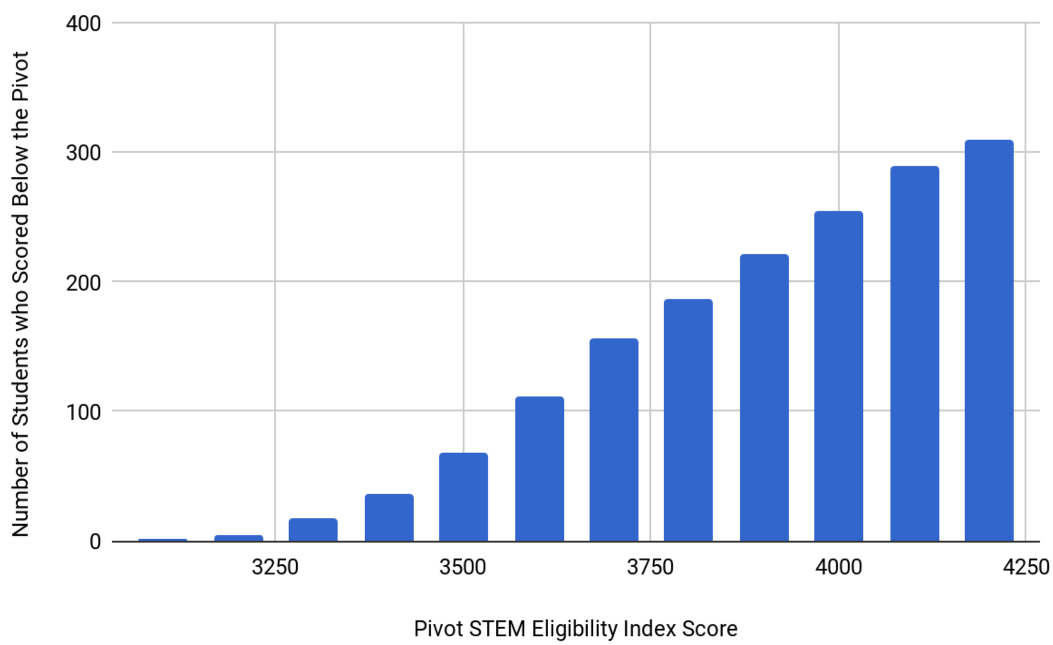

Figure 10. *Number of students (vertical axis) with STEM Eligibility Index Scores (horizontal axis) below each pivot studied — numbers between 3100 and 4200, inclusive, increment by 100.*

Failure Rates in STEM Eligibility Index Score Ranges

Table 6 includes computed Math 113 failure rates over ten STEM Eligibility Index score ranges. The expectation for this analysis was that failure rates would decrease as STEM Eligibility Index scores increased. Overall, the data demonstrates this trend with the exception of 3500-3700. Based on the data, students who fail to achieve a STEM Eligibility Index score of 3300 are likely to fail Math 113 with a probability of 0.5882.

STEM Eligibility Index Score Range	Failure Rate	Failure Rate Expressed as a Percentage	Number of Students with STEM Eligibility Index Scores in the Range
0 - 3300	0.5882	58.82	17
3300 - 3400	0.5789	57.89	19
3400 - 3500	0.5625	56.25	32
3500 - 3600	0.4318	43.18	44
3600 - 3700	0.4545	45.45	44
3700 - 3800	0.2258	22.58	31
3800 - 3900	0.1714	17.14	35
3900 - 4000	0.0909	9.09	33
4000 - 4100	0.0882	8.82	34
4100 - 4800	0.0313	3.13	32

Table 6. *Results of failure rate analysis in STEM Eligibility Index score ranges. Data from the 321 freshmen Math 113 students used in the study was divided into 10 categories based on students' STEM Eligibility Index scores. Within the STEM EI score ranges, Math 113 failure rates were calculated. The expectation was that failure rates would decrease as STEM EI scores increased.*

Figures 11 and 12 are graphical representations of the information presented in Table 6.

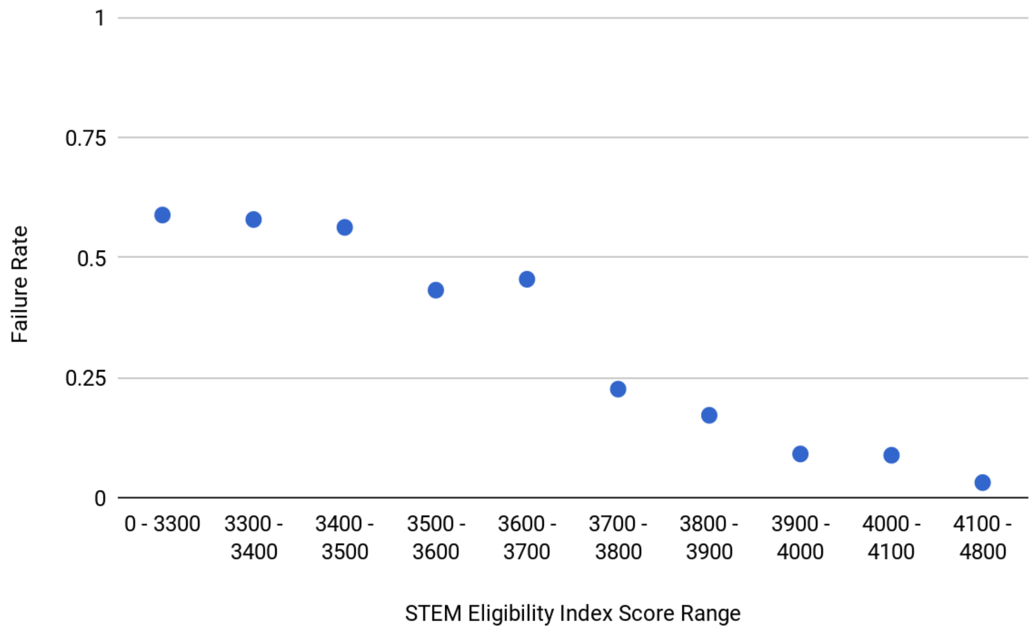

Figure 11. *Graphical representation of the results of failure rate analysis in STEM Eligibility Index score ranges. Math 113 failure rates (vertical axis) generally decrease as STEM Eligibility Index scores (horizontal axis) increase, confirming the expectation.*

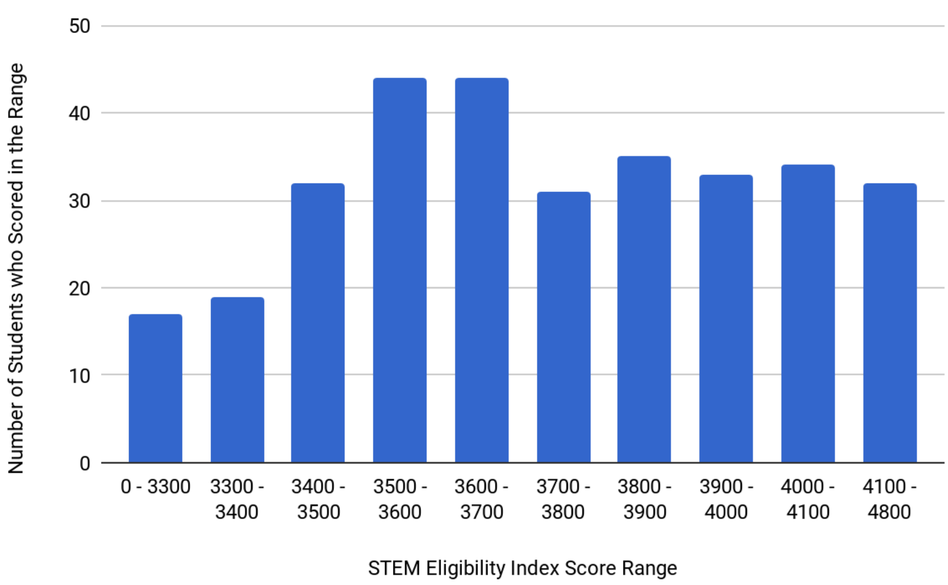

Figure 12. *Number of students with STEM Eligibility Index scores in each score range. Sample sizes are small but similar across score ranges between 3400 and 4100, inclusive, indicating a fairly even distribution of data in this analysis.*

Logistic Regression

The computed coefficients for the logistic regression model were as follows:

α = -14.7396

β_1 = 0.3459

β_2 = 0.0340

β_3 = 0.0034

Thus the resulting regression model for this study was:

Logit [P(Y)] = -14.7396 + 0.3459(GPA) + 0.0340(ALEKS Score) + 0.0034(STEM EI Score)

GPA

β_1 = 0.3459 indicates that a one unit change in GPA (e.g. 2.5 to 3.5 or 3.0 to 4.0) corresponds to a 0.3459 unit change in log odds of passing Math 113. Thus if ALEKS score and STEM Eligibility Index score are held constant, a student who demonstrates a one unit increase in GPA will increase their odds of passing Math 113 by 1.4132 times. A one unit increase in GPA however is quite sizeable, so it would be more practical for the purposes of this study to examine the effects of a 0.25 unit increase in GPA. A 0.25 unit change in GPA (e.g. 3.25 to 3.50 or 3.50 to 3.75) corresponds to a 0.0865 unit change in the log odds of passing Math 113. Therefore, all else held constant, a 0.25 unit increase in GPA leads to a student's odds of passing Math 113 becoming 1.0903 times greater. Equivalently, by increasing GPA by 0.25 units, a student becomes approximately 9% more likely to pass Math 113.

ALEKS Score

β_2 = 0.0340 indicates that a one unit change in ALEKS score (e.g. 46 to 47 or 59 to 60) corresponds to a 0.0340 unit change in the log odds of passing Math 113. Thus if a student increases his or her ALEKS score by one unit, he or she will become 1.0346 times more likely to pass Math 113. If however a student spends sufficient time in the remediation module, he or she should experience an increase in his or her ALEKS score that is much larger than one unit. In CSULB's freshman class of 2017, twenty-one students who participated in the Early Start Math Program placed into Math 113. Among that sample of students, the average increase in ALEKS score from initial assessment to highest assessment was 20.429 points. Based on the results of the logistic regression, a 20 unit change in ALEKS score corresponds to a 0.6793 unit change in the log odds of passing Math 113. Thus, all else held constant, with a 20 unit increase in ALEKS score, a student becomes 97.24% more likely to pass Math 113.

STEM Eligibility Index

β_3= 0.0034 indicates that a one unit change in STEM Eligibility Index score (e.g. 3300 to 3301) corresponds to a 0.0034 unit change in the log odds of passing Math 113. Thus a one unit increase in STEM Eligibility Index will increase a student's odds of passing Math 113 by 1.0034 times. A one unit change in STEM Eligibility Index is by no means significant considering the formula used to obtain the STEM Eligibility Index score: (College Preparatory Grade Point Average x 600) + SAT Critical Reading + (2 x SAT Math). A one unit change can result from a 0.002 unit rounding error in GPA. Thus a larger change in STEM EI must be examined in order to produce meaningful insight. The impacts of a 200 unit change in STEM Eligibility Index Score will be presented for reasons that will be discussed later in the Discussion and Recommendations section. According to the results of the regression analysis, all else held constant, if a student displays a 200 point increase in STEM EI score, they become 95.75% more likely to pass Math 113.

Discussion and Recommendations

Appropriate ALEKS and STEM Eligibility Index Cut Scores

The first two questions of this study were as follows: What cut scores for the ALEKS mathematics assessment and STEM Eligibility Index would result in successful placement of students in pre-technical majors in Math 113? Does the current ALEKS cut score of 46 for Math 113 produce the expected pass rate for that course?

The results listed in Table 1 indicate that the probability of a student passing Math 113 given that their ALEKS assessment score is above a particular pivot generally increases as the pivot score increases. Thus the data used in this study supports the expectation for the conditional probability analysis. At the pivot score of 46, the current ALEKS cut score for Math 113 at CSULB, it can be observed that students who meet or exceed that score have a 75.28% chance of passing Math 113 which is a fairly strong probability; however, in order to provide further insight, failure rates must be examined. From Table 3, it is known that students in the freshman class of 2017 who scored in the range of 46-48 on the ALEKS assessment had a 31.58% failure rate in Math 113. Unfortunately, these results cannot be applied to make predictions because they were obtained from a small sample size of 19, and therefore are somewhat unreliable.

Examining the probability that a student receives a failing grade in Math 113 given that the student scores below a particular pivot provides further information where the failure rate analysis fails to do so. At the current cut score of 46, it can be observed that students who score below this pivot on the ALEKS assessment have a 59.26% chance of failing Math 113. When the pivot increases from 46 to 48, the conditional probability of failure decreases by 4.85 percentage points to 54.41%, and it decreases by another 5.04 percentage points to 49.37% when the pivot increases from 48 to 50. Overall, a 4 point increase in ALEKS assessment scores produces a 9.89 percentage point decrease in conditional probability of failure.

Given that a student who attains an ALEKS score of 46 or greater has a 75.28% chance of passing Math 113, I would not recommend increasing the cut score based on the conditional probability analysis of failure, but I would recommend taking additional information into consideration for students with ALEKS scores between 46 and 50, namely the student's STEM Eligibility Index score.

Like ALEKS scores, STEM Eligibility Index scores met the expectation that the conditional probability of passing Math 113 would increase as pivot scores increased (see Table 4). CSULB currently requires that applicants for STEM majors have a STEM Eligibility Index score of at least 3300. The results of this study show that the conditional probability of passing Math 113 given that a student meets or exceeds this STEM EI score is 71.05%. If the 3500 pivot is examined, the respective probability increases by 5.63 percentage points to 76.68% which exceeds a ¾ chance of passing Math 113.

The data in Table 6 indicates a sizeable decrease in failure rates over the 3300 to 3500 STEM EI score range. Students in the freshman class of 2017 who had a STEM Eligibility Index score between 3300 and 3400 failed Math 113 at a rate of 57.89%. When the score range under examination is increased to 3500-3600, the failure rate decreases by 14.71 percentage points to 43.18%.

Because both the conditional probability analysis and the failure rate analysis show a notable increase in probability of passing Math 113 when STEM Eligibility Index scores increase from 3300 to 3500, I would recommend requiring that students with borderline ALEKS scores, scores in the 46-50 range, have a STEM Eligibility Index score of at least 3500 in order to enroll in Math 113.

The STEM Eligibility Index score requirement of 3500 for students with ALEKS scores between 46 and 50 may also work to combat a performance gap that could potentially arise between students placed in Math 113 based on GPA and SAT/ACT scores and students placed in Math 113 based on ALEKS scores. Come Fall 2018, there will be three possible paths through which students can be placed into Math 113: (1) student has a high school GPA greater than or equal to 3.50 and an old SAT Math score equivalent of 500 or greater, (2) student has an old SAT Math score equivalent of 570 or greater, or (3) student has an ALEKS Assessment score of 46-69 (CSULB, n.d.).

According to the STEM Eligibility Index formula, students with a high school GPA of 3.50 and an SAT Math score of 500 have a STEM EI score of 3100 before the Critical Reading portion of the SAT is factored in. Considering the fact that the SAT Critical Reading section is scored on a scale of 200-800, the students is guaranteed to meet the minimum STEM EI score requirement of 3300 and very likely to reach the 3500 threshold. Similarly students with an SAT Math score of 570 and a GPA of 3.25 (note this GPA is 0.25 units lower than the 3.50 GPA required by path 1) will have a STEM EI score of 3090 before the SAT Critical Reading score is factored in. Thus they too are likely to meet or exceed the STEM EI score threshold of 3500. Thus students placed in Math 113 through the first two paths will likely enjoy the statistical increases in conditional probability of passing Math 113 and statistical decreases in failure rates for students with a STEM EI score greater than or equal to 3500.

High School GPA, ALEKS Scores, and STEM Eligibility Index as Predictors of Success in Math 113

The final question of this study was: What impacts do high school GPA, performance on ALEKS assessments, and STEM Eligibility Index score have on a student's probability of passing Math 113? Based on the results of the logistic regression, the change in odds of passing Math 113 associated with a specific change in GPA, ALEKS score, or STEM Eligibility Index can be quantified.

As previously stated, an increase in ALEKS score from 46 to 50 makes a notable 9.89 percentage point difference in the conditional probability that a student will fail Math 113 given that their score is below the pivot. The logistic regression model shows that the 4 unit increase required to take the ALEKS score from 46 to 50 correlates with a student's odds of passing Math 113 becoming 1.1455 times greater. This difference in odds over a 4 point range reinforces the assertion that students with an ALEKS score in the lower end of this range are at greater risk of failure and thus should be further evaluated based on their STEM EI score to ensure that they will be successful in Math 113.

The recommendation made was that students who are in the borderline region for ALEKS scores should have a STEM Eligibility Index score of 3500, rather than the minimum 3300, in order to enroll in Math 113. Conditional probability analyses revealed that given a student has an ALEKS score of at least 50 or a STEM EI score of at least 3500, he or she will have at least a 75% chance of passing Math 113. As discussed in the Results section, a 200 point increase in STEM EI score is associated with a student's odds of passing Math 113 becoming 1.9575 times greater. Thus a student with an ALEKS score of 46 and a STEM EI score of 3500 would be almost two times more likely to pass Math 113 than a student with an ALEKS score of

46 and a STEM EI score of 3300. This example alone demonstrates the importance of taking STEM EI scores into consideration for students who are on the margin for placements decisions.

In this study, high school GPA was used as a proxy for students' study skills. As shown in the Results section, a 0.25 point increase in GPA correlates with a student's odds of passing Math 113 becoming 1.0903 times greater. Thus while high school GPA offers valid information about a student's odds of passing Math 113, it is not as useful as STEM Eligibility Index in making placement decisions for students with borderline ALEKS scores.

Conclusion

Given that a student achieves CSULB's current ALEKS cut score of 46, he or she will have a 75.28% chance of passing Math 113. If a student reaches the minimum STEM Eligibility Index score of 3300, he or she will have a 71.05% chance of passing Math 113. Based on these probabilities, it is reasonable to state that the current ALEKS cut score and STEM Eligibility Index requirement at California State University Long Beach produce a reasonable pass rate for Math 113 students. The most notable recommendation of this study is that STEM Eligibility Index should be taken into consideration for students with ALEKS scores just above the cut score, between 46 and 50, by requiring such students to have a STEM EI score of at least 3500.

Directions for Future Research

As previously stated, come Fall 2018, the California State University system will be eliminating remedial math courses for all twenty-three campuses. In response, California State University Long Beach has designed a Precalculus Algebra course sequence that will cover the same material as Math 113 over the course of two semesters. The course will be listed in the course catalog as Math 112A and Math 112B. Students in pre-technical majors who fail to meet the requirements for placement into Math 113 will enroll in this course sequence.

After the change in course offerings, a similar study should be conducted to determine the appropriate ALEKS cut score and STEM Eligibility Index score to prevent overplacement in Math 113 and underplacement in Math 112A and Math 112B.

It was also noted that the failure rates over ALEKS score ranges found in this study could not be used to make reliable predictions due to small sample sizes. With the use of the ALEKS PPL in place of the CSU system-wide math placement exam, more students will have ALEKS scores and thus sample sizes will increase, allowing for useful predictions to be drawn. Therefore, the study should be repeated on a future freshman class in order to extract useful conclusions regarding ALEKS scores and failure rates.

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