Illumination Face Spaces are Idiosyncratic

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Illumination variations - movie

Illumination variations

How do others commonly handle illumination variations?

Illumination normalization

Single to single image comparison

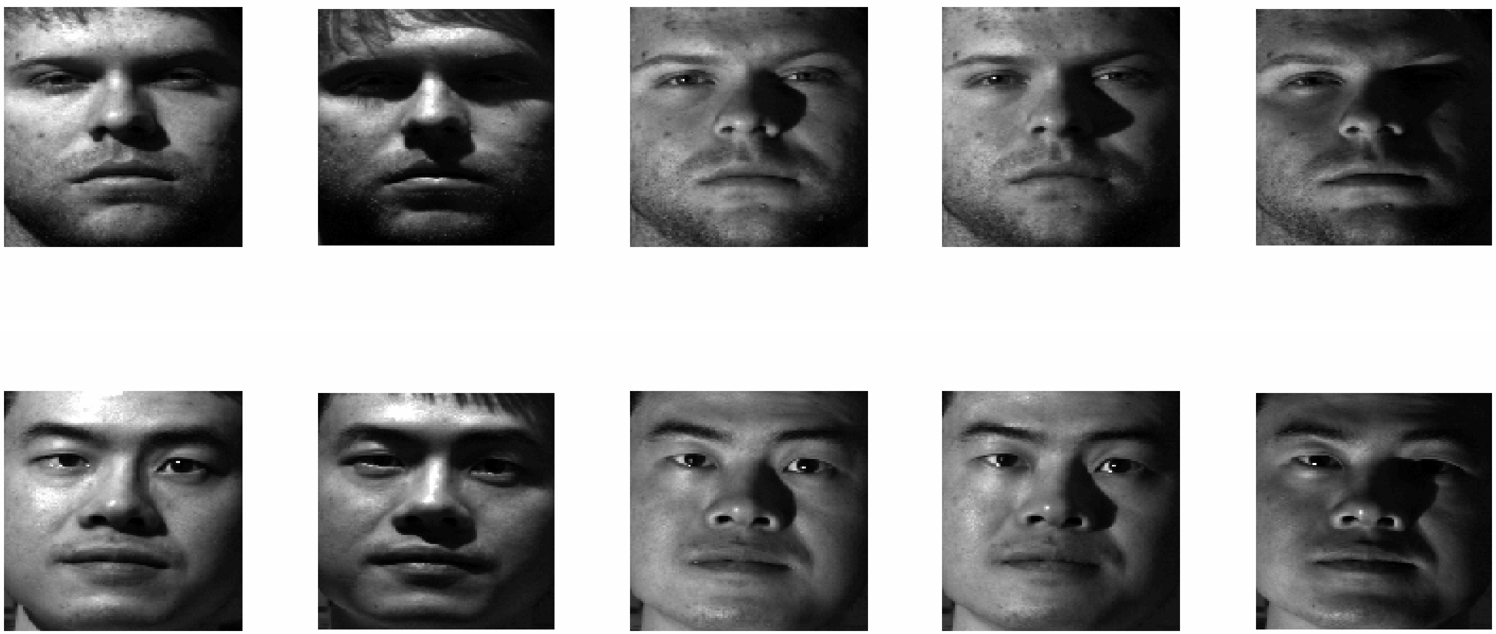
Can you tell them apart?

Single to single image comparison

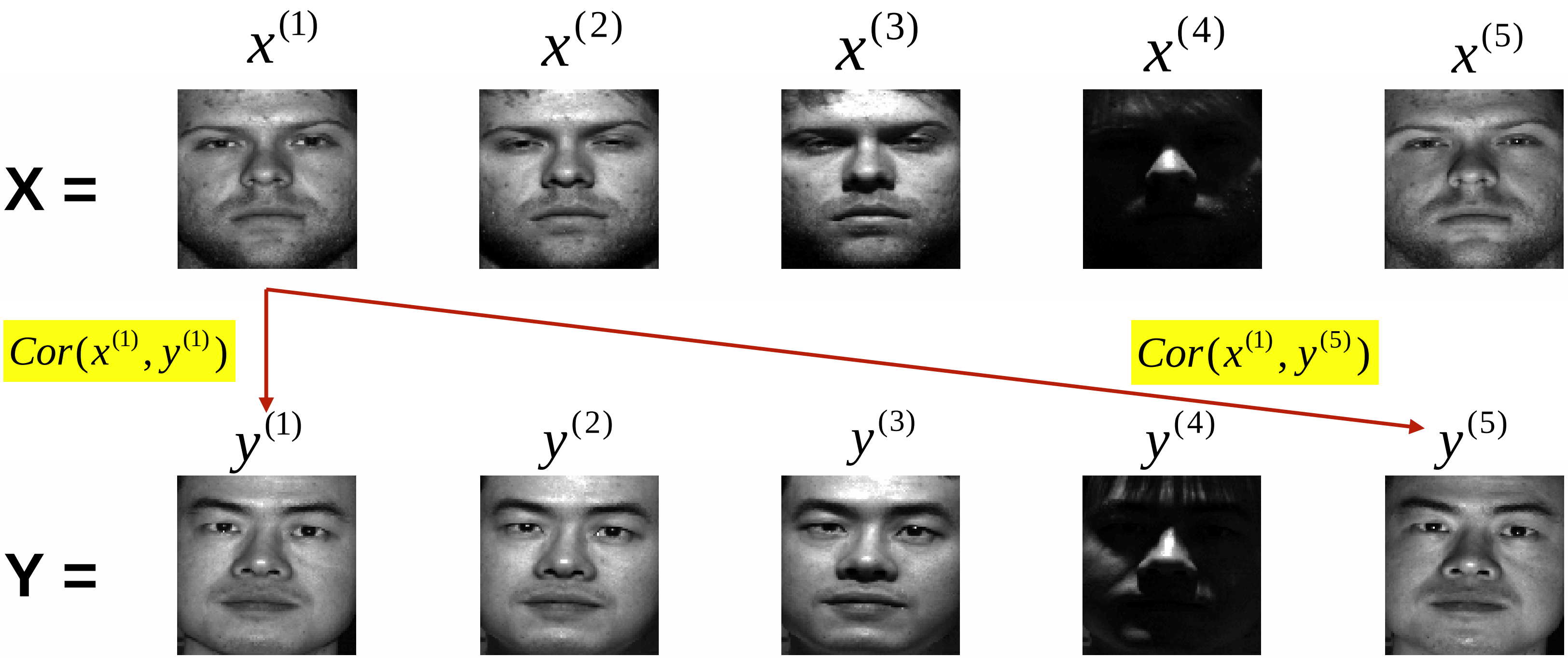
Can you tell them apart?

Multi-still to multi-still image comparison

Can you tell them apart?



Comparison based on correlations – a baseline algorithm

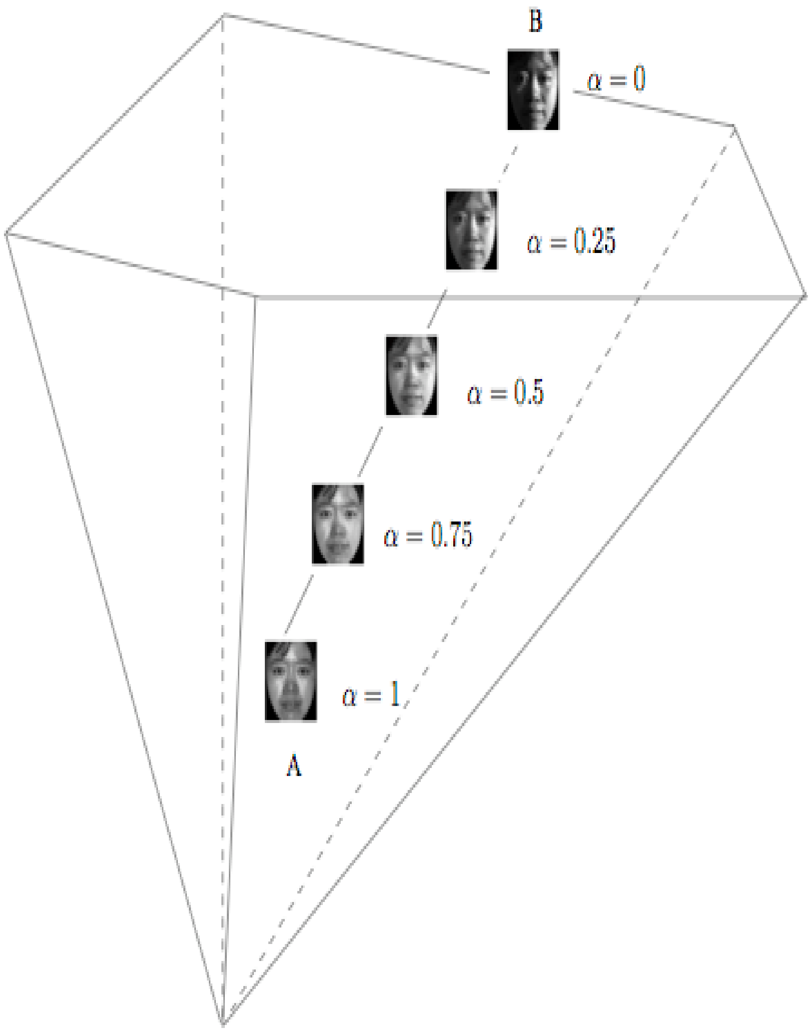


Similarity score for comparing multi-still sets X and Y is defined to be,

$$S(X,Y) = \frac{1}{2}\sum_{j=1}^{k}\left(s(x^{(j)},Y) + s(y^{(j)},X)\right)$$, where $s(x^{(j)},Y) = \max_{1\le i\le k} \{Cor\,(x^{(j)}, y^{(i)})\}$

for all $x^{(j)} \in X$ and $y^{(j)} \in Y$

Illumination space - geometry



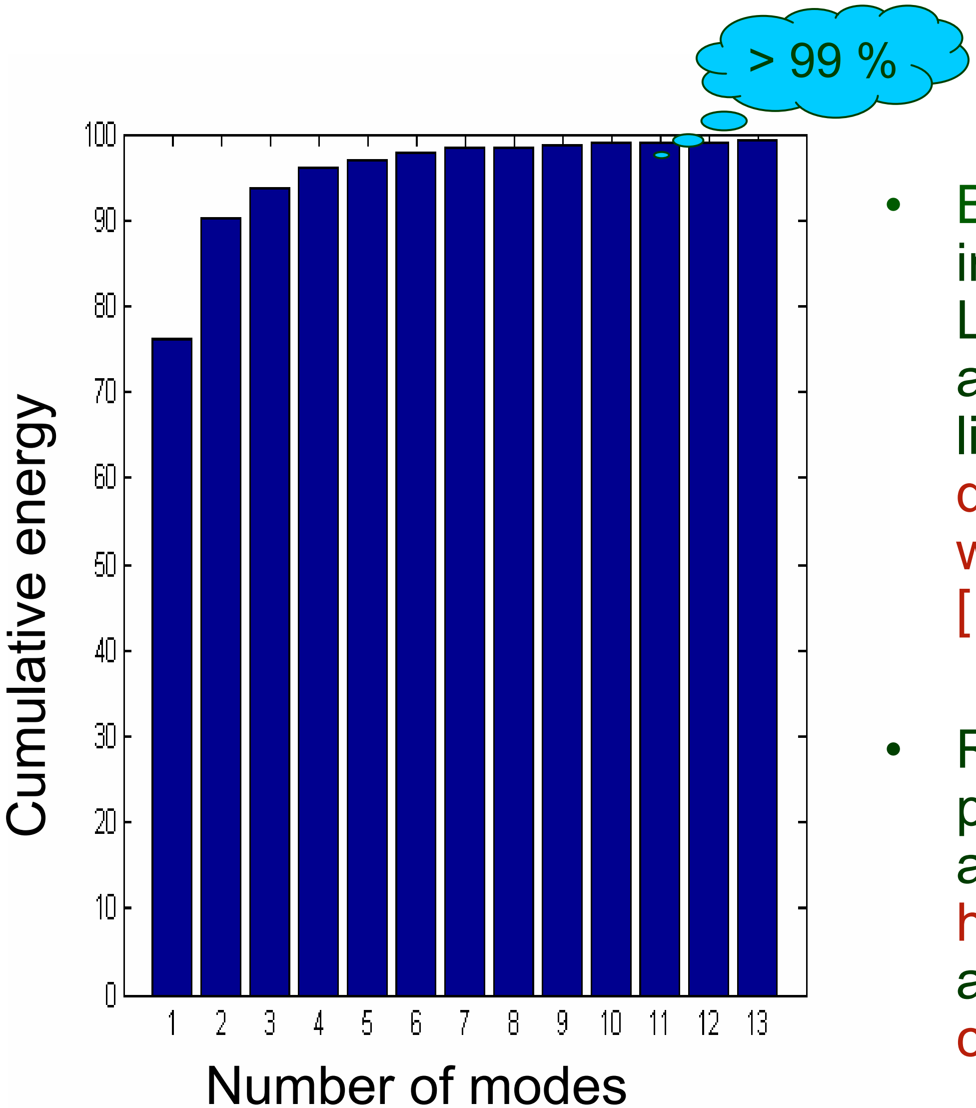
- **Belhumeur and Kriegman** – the set of n-pixel monochrome images of an object of any shape with a general reflectance function, seen under all possible illumination conditions, forms a convex polyhedral cone [2].
- If A and B are in an illumination cone C, then for all $\alpha \in [0,1]$, α A + (1- α) B $\in$ C.

Cone convexity - movie

A

B

Illumination space - geometry

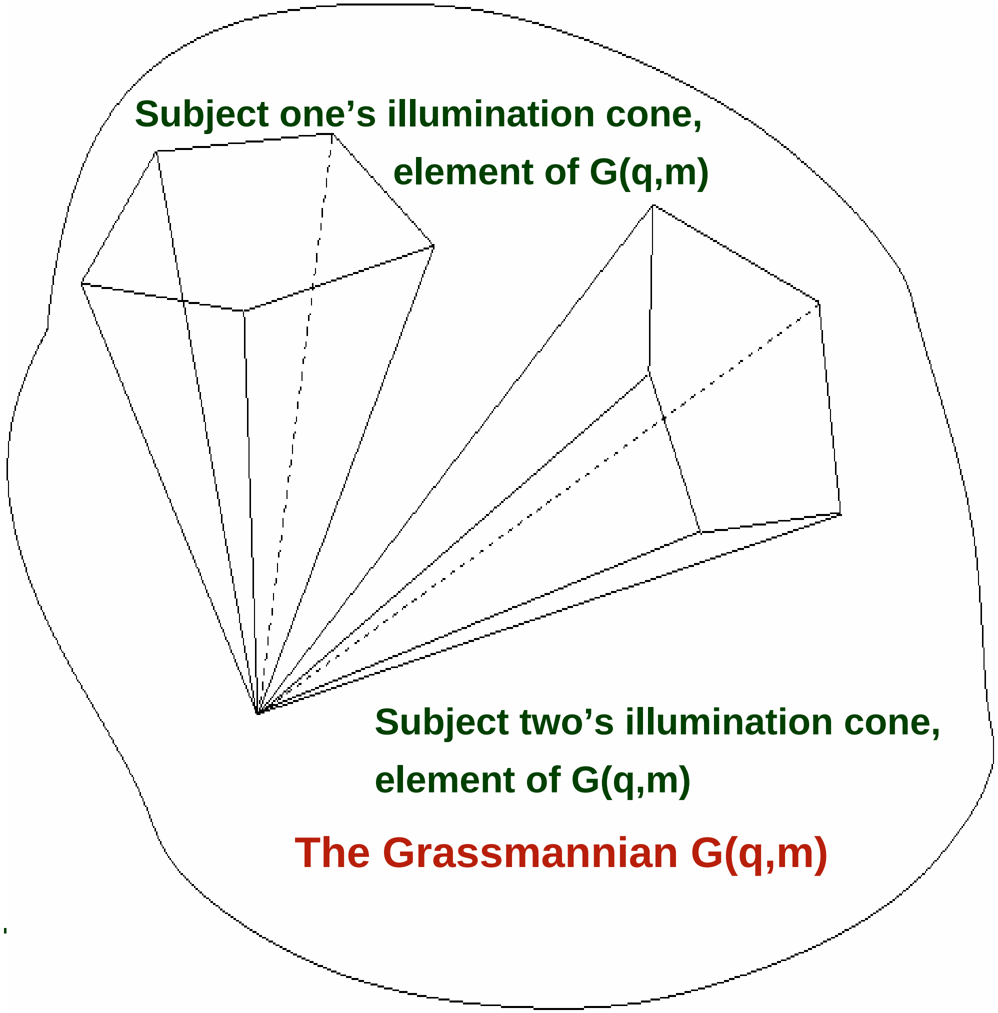


- Basri and Jacobs – the set of images of a convex, Lambertian object seen under arbitrary distance light sources lies approximately in a 9-dimensional linear subspace with over 99% of the energy [1].

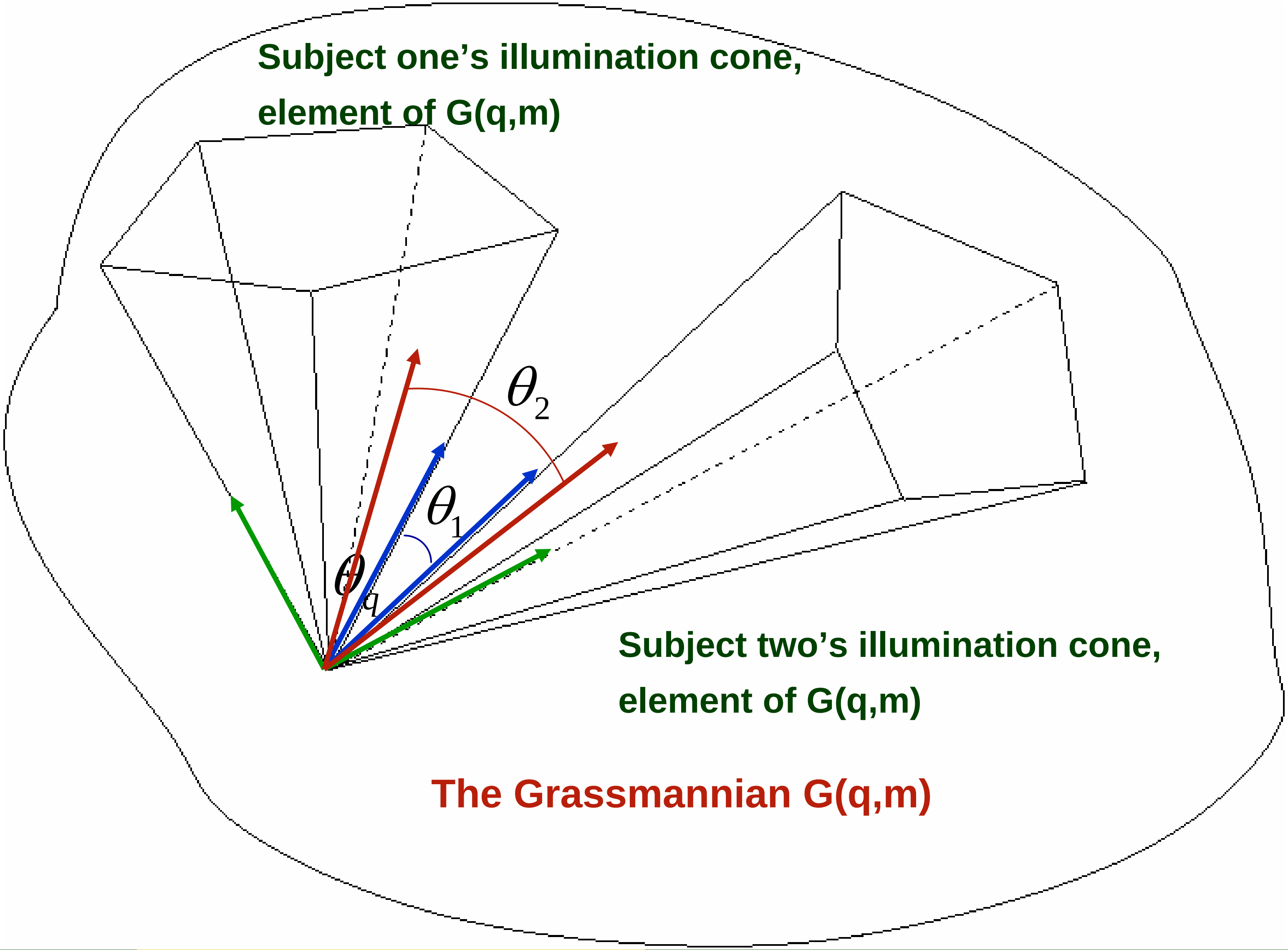
- Ramamoorthi – Transforms the problem of linear approximation with spherical harmonics into linear approximation with principal components [8].

Illumination spaces and the Grassmannian

- We model illumination spaces as points on a geometric object known as the Grassmann manifold or the Grassmannian.
- A Grassmannian G(q,m) is a m-dimensional geometric object whose points parameterize subspaces of a fixed dimension, q.
- We measure the distance between subspaces by examining the principal angles.



Principal angles - idea



Principal angles - definition

If X and Y are two vector subspaces of $\mathbb{R}^m$, then the *principal angles* $\theta_k \in [0, \frac{\pi}{2}]$, $1 \leq k \leq q$ between X and Y are defined recursively by

$$\cos(\theta_k) = \max_{u \in X} \max_{v \in Y} u^T v = u_k^T v_k$$

subject to $||u|| = ||v|| = 1$, $u^T u_i = 0$ and $v^T v_i = 0$ for $i = 1 : k-1$ and $q = \min\{\dim(X), \dim(Y)\} \geq 1$.

- Thus, at the end of the comparison between two subspaces, we obtain a vector of principal angles $\theta = (\theta_1, \theta_2, \ldots, \theta_q)$ that tell us the geometric relationship between the two subspaces.

Idea of principal angles – optimization by deflation

$$u_1 = \sum_{i=1}^{q} \alpha_i x_i, \alpha_i = scalar$$

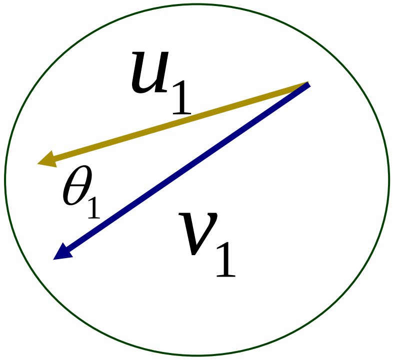
$$v_1 = \sum_{i=1}^{q} \beta_i y_i, \beta_i = scalar$$

Subspace of subject 1

Basis (i)	Vector (x_i)
1	→
2	→
3	←
⋮	↘
q	↖

Subspace of subject 2

Basis (i)	Vector (y_i)
1	↖
2	↓
3	→
⋮	→
q	←



$$\cos(\theta_1) = \frac{u_1^T v_1}{\|u_1\| \|v_1\|}$$

Should we consider…?

Single?

$$(\theta_1 = \theta_{\min})$$

$$(\theta_q = \theta_{\max})$$

$$(\theta_i)$$

OR

Multiple?

$$(\theta_1, \theta_2)$$

$$(\theta_1, \theta_2, \theta_3)$$

$$(\theta_1, \theta_2, \theta_3, \ldots, \theta_\ell)$$

It is revealing to consider nested subspaces of X and Y in G(q,m) by defining the ℓ - truncated principal angle vector $\theta^\ell := (\theta_1, \theta_2, \ldots, \theta_\ell)$, where $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_\ell$ are the principal angles between X and Y and $1 \leq \ell \leq q$.

Truncated Grassmannian distances

Arc Length (Geodesic) [4]	$d_g^\ell(X,Y) = \left\|\theta^\ell\right\|_2$
Fubini-Study [7]	$d_{FS}^\ell(X,Y) = \cos^{-1}\left(\prod_{i=1}^{\ell}\cos\theta_i\right)$
Projection F (Chordal) [3]	$d_c^\ell(X,Y) = \left\|\sin\theta^\ell\right\|_2$
Chordal Frobenius	$d_{cF}^\ell(X,Y) = \left\|2\sin\frac{1}{2}\theta^\ell\right\|_2$
Subspace Distance [6]	$d_{ss}^\ell(X,Y) = \left\|\sin\theta^\ell\right\|_\infty$

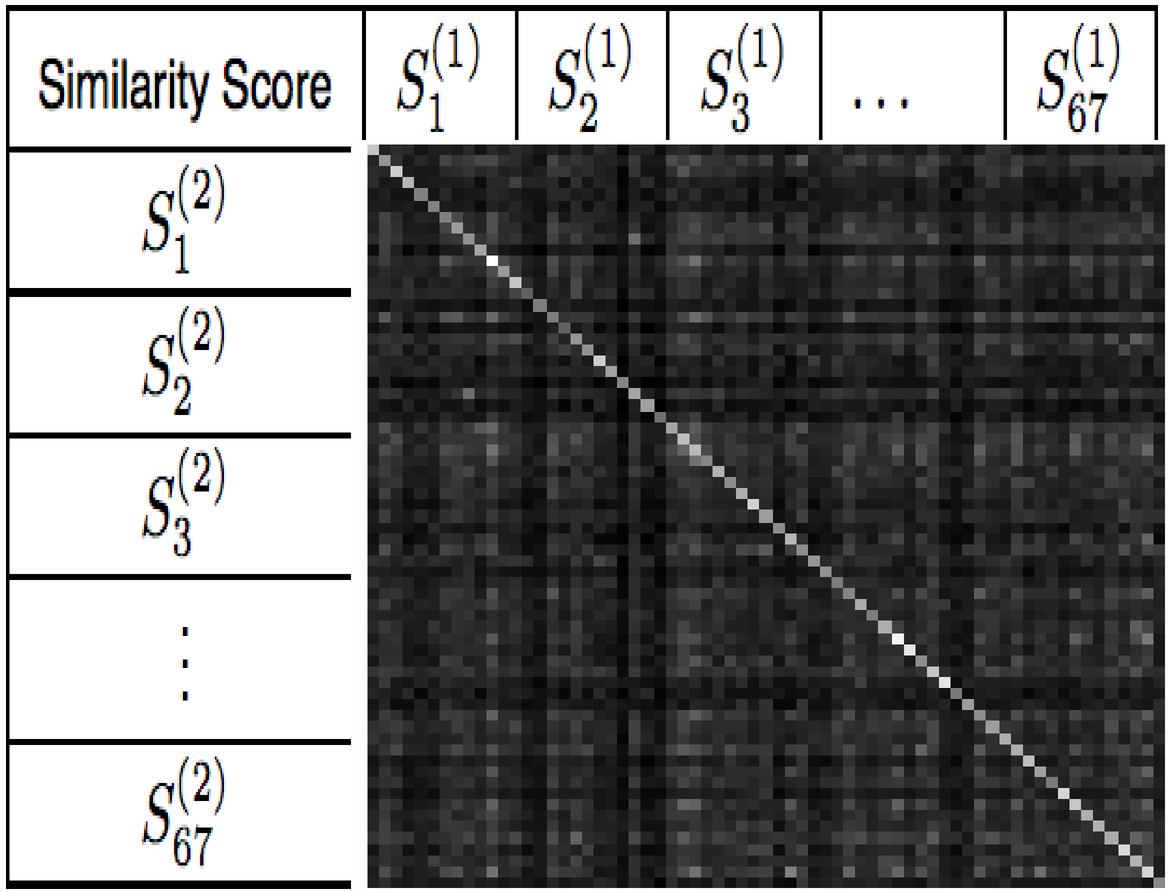
Concept of idiosyncrasy

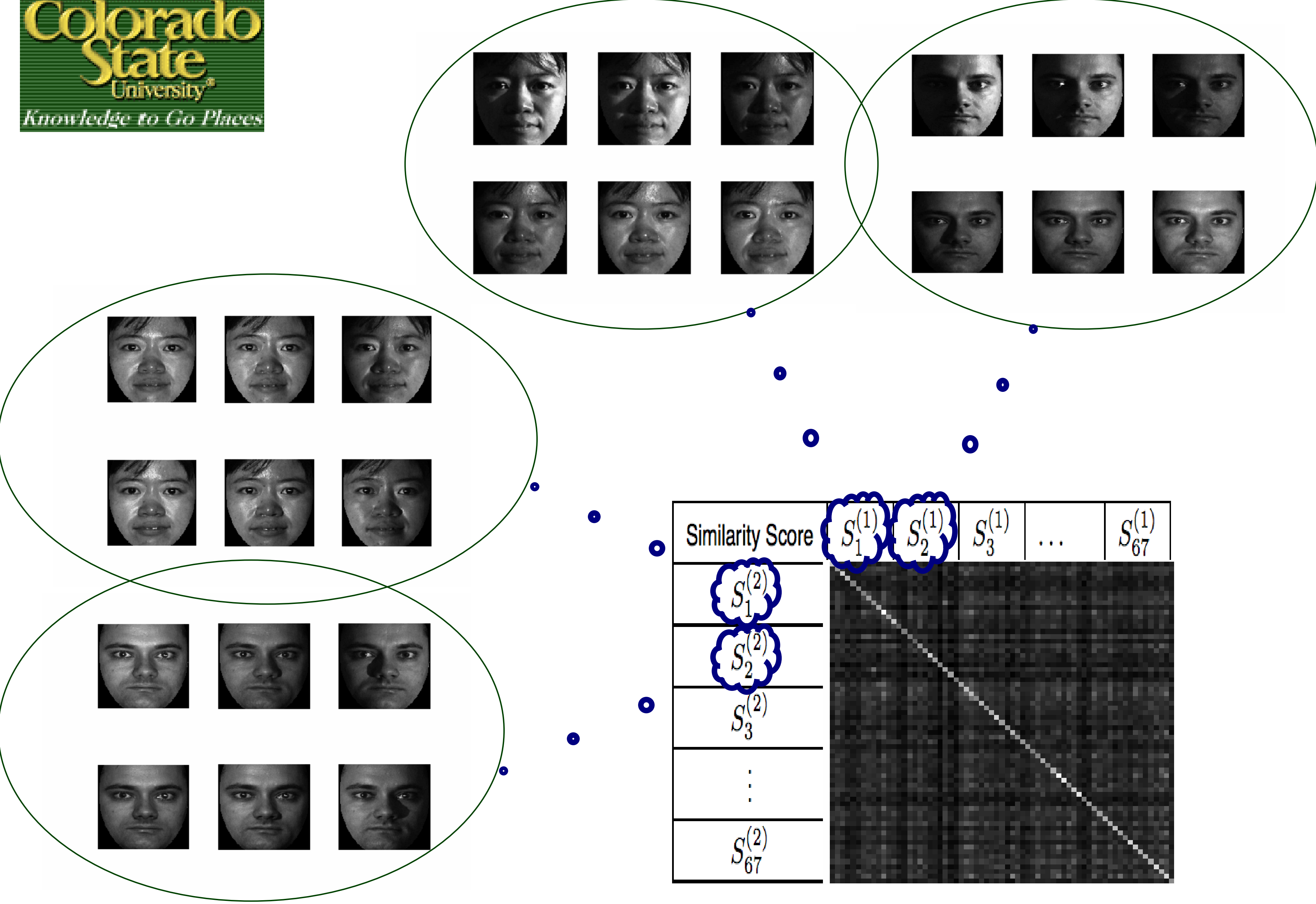
An illumination space estimated from images of one subject should always be "closer" to another illumination space estimated from images of the same subject than to any illumination space estimated from images of a different subject.

Methods of experimentation

- We estimate two illumination subspaces for every subject in the Yale [5] and CMU-PIE [9] data sets.
- The subspaces for each person are estimated from randomly selected sets of 8 or more images of the subject.
- For the 67 subjects in the CMU-PIE data set, this creates 67 pairs of matching subspaces and 4,422 pairs of non-matching subspaces.
- For the 10 subjects in the Yale Database B, this creates 10 pairs of matching subspaces and 90 pairs of non-matching subspaces.

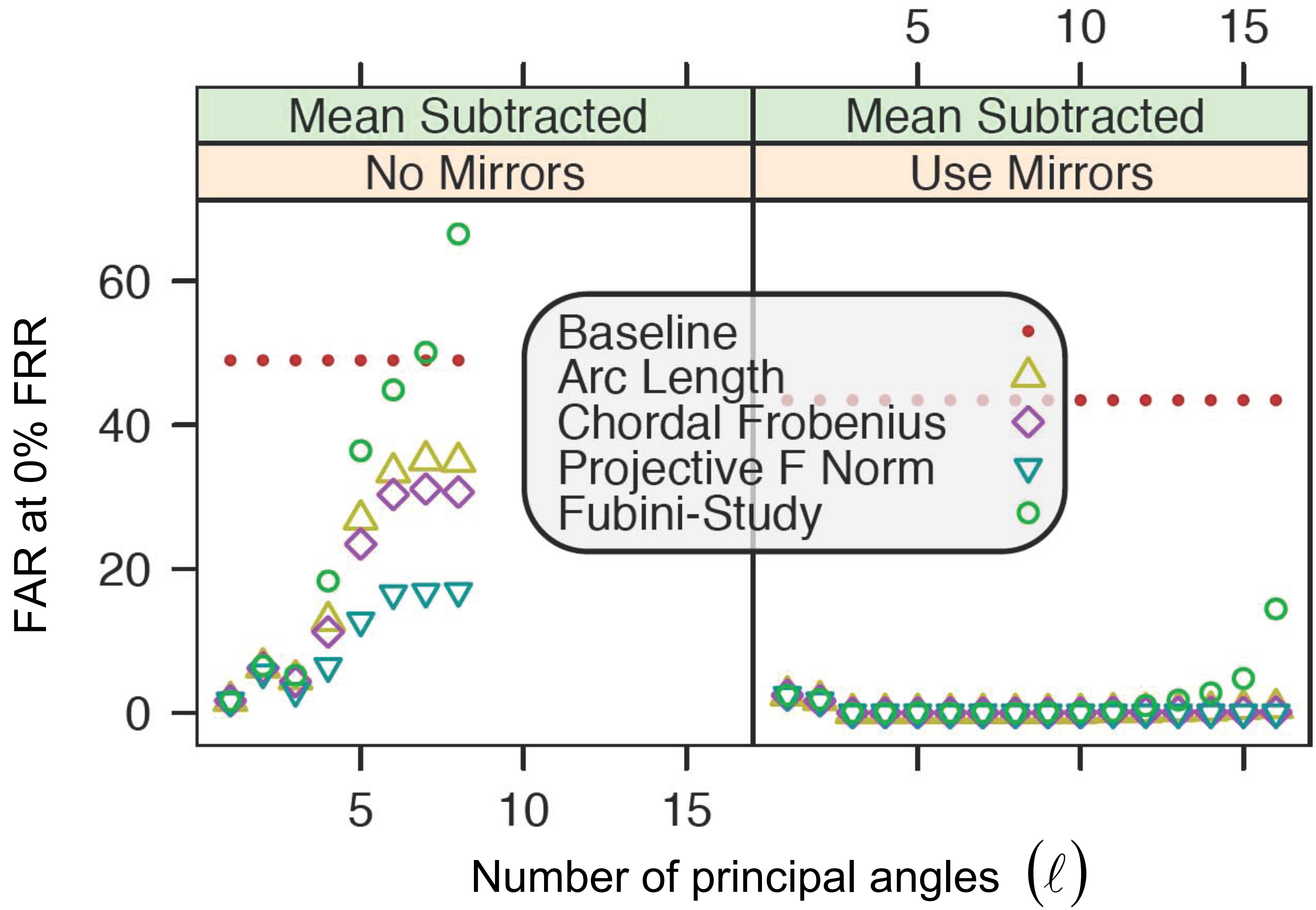
Data structure



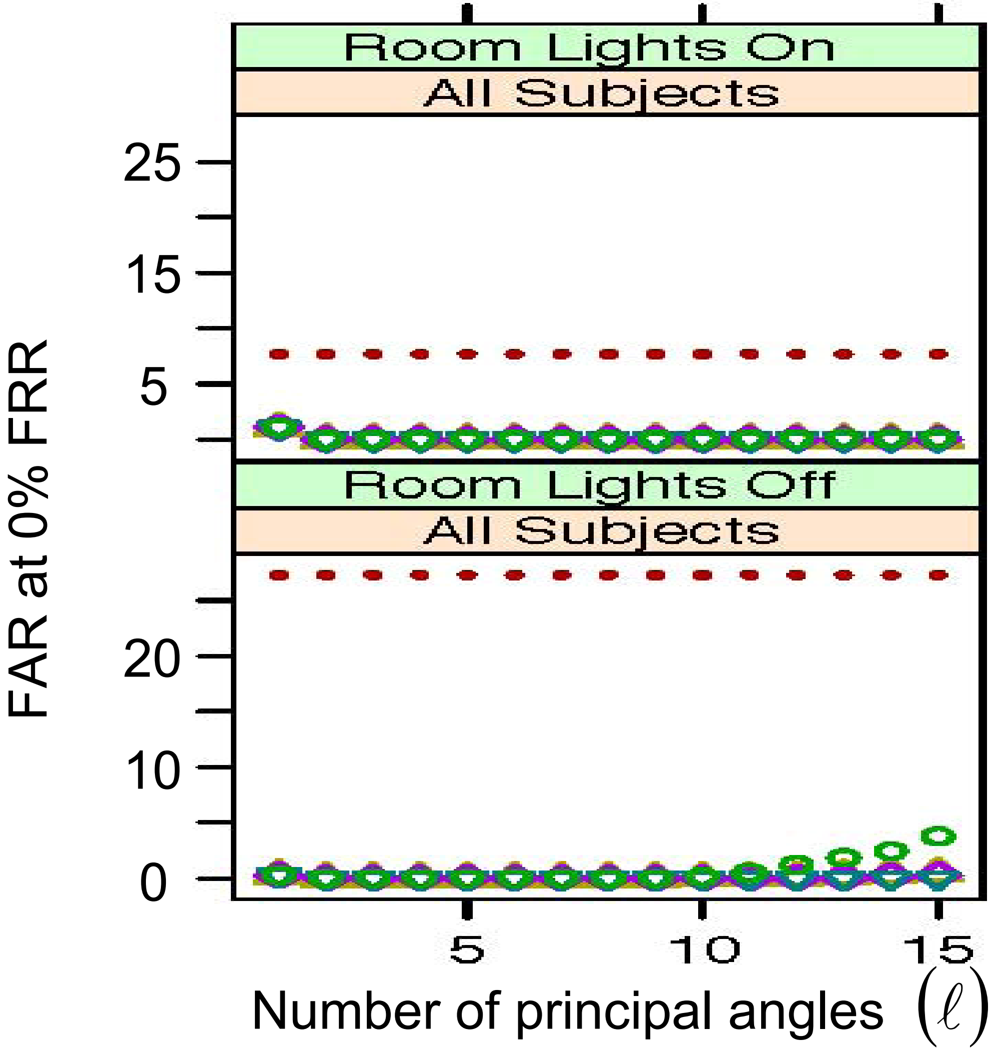


Similarity Score
$S_1^{(1)}$ $S_2^{(1)}$ $S_3^{(1)}$ … $S_{67}^{(1)}$
$S_1^{(2)}$
$S_2^{(2)}$
$S_3^{(2)}$
⋮
$S_{67}^{(2)}$

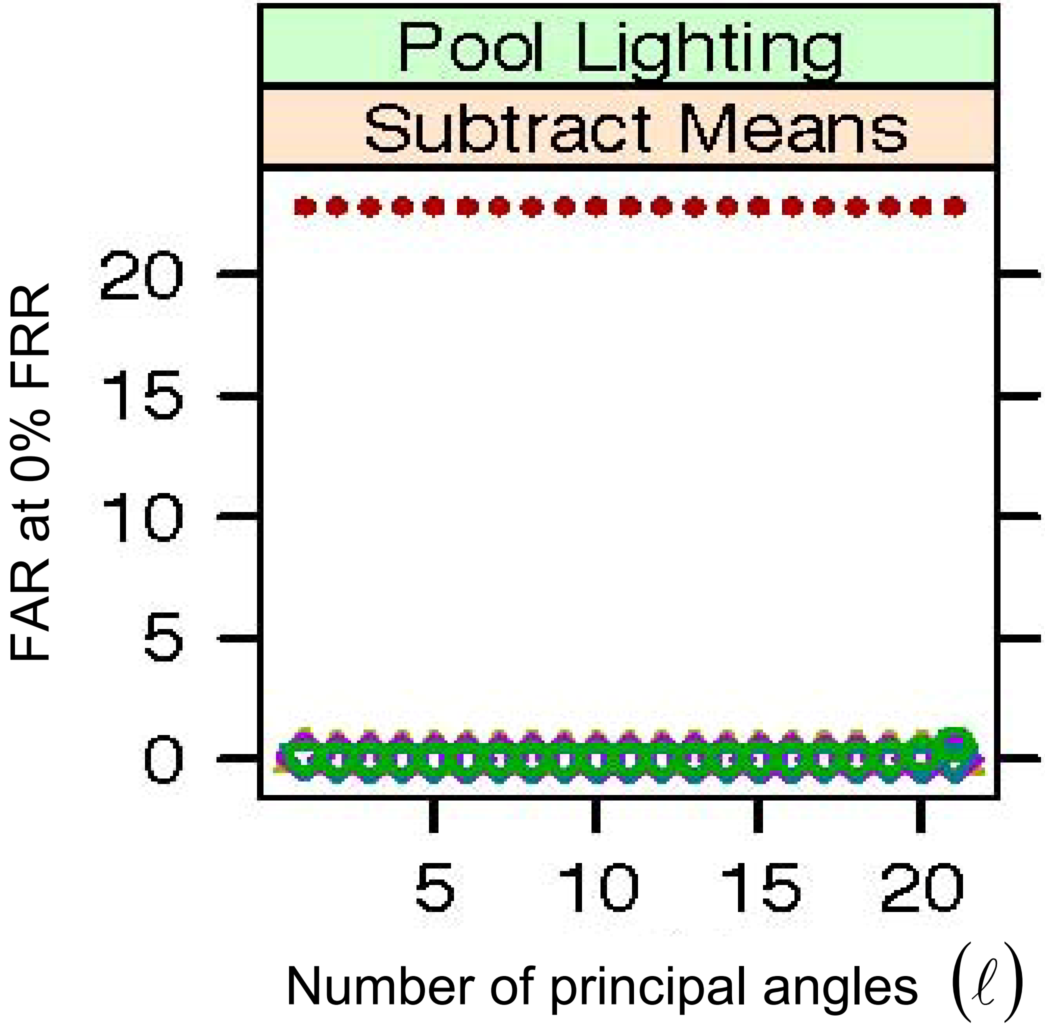
Empirical results – YDB



Empirical results – CMU-PIE, separate lighting conditions



Empirical Results – CMU-PIE, pooled lighting conditions



Analysis of empirical results

- For each of the ten trials and all 67 subjects in CMU-PIE data set, the distance between the two matching subspaces is always less than the distance between any non-matching pair of subspaces.
- For each of the ten trials and all 10 subjects in Yale Database B, the distance between the two matching subspaces is always less than the distance between any non-matching pair of subspaces.
- We therefore assert that these data sets are Grassmann separable and illumination face spaces are idiosyncratic.

Conclusions – 1/4

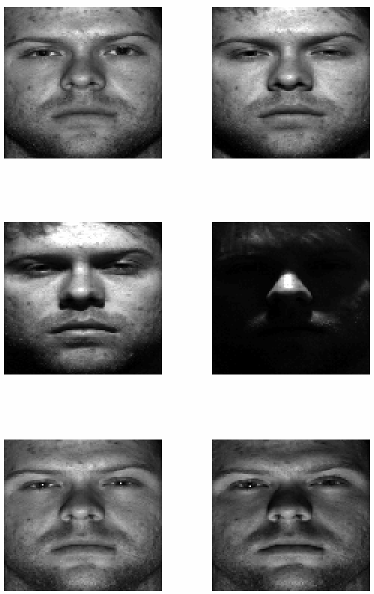
= Feature

≠ Noise

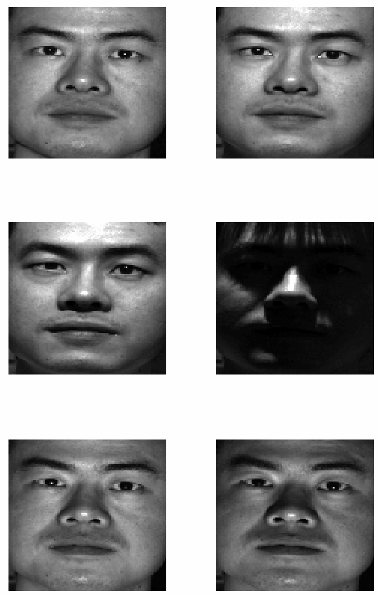
Conclusions – 2/4

V.S.

=



V.S.



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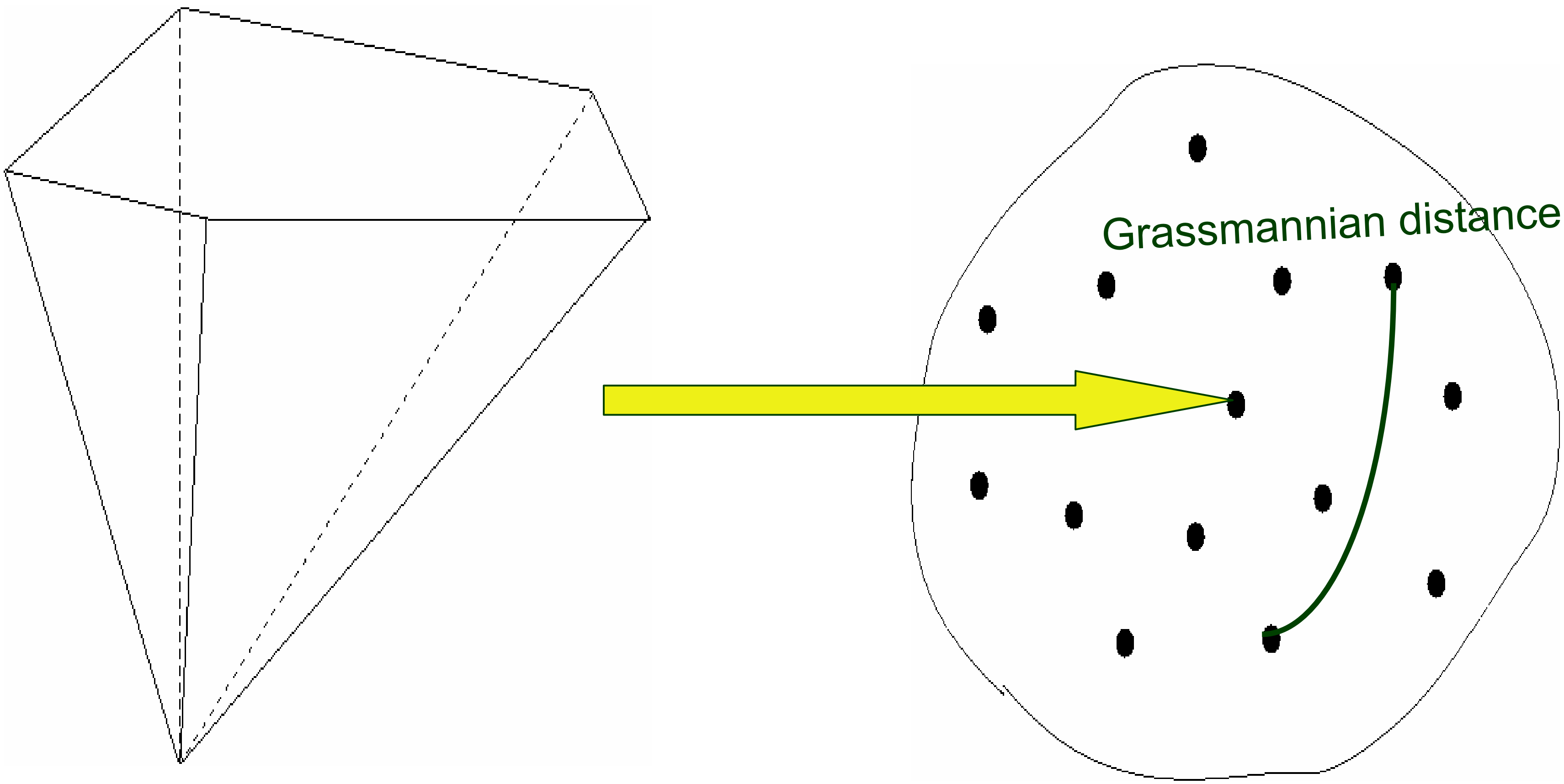
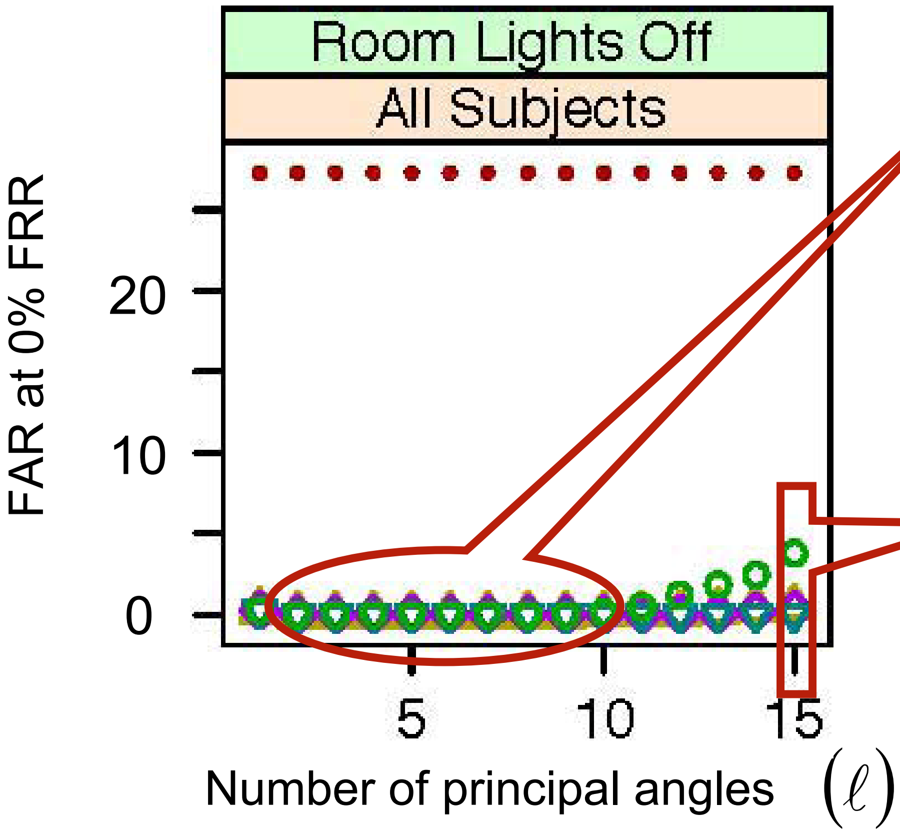
Conclusions – 3/4

An illumination space = a point on a Grassmann manifold



Conclusions – 4/4



Perfect separation when considering truncated principal angles

Imperfect separation when using ALL of the principal angles available

- Thank you for your attention -

References

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