



**Increase student learning
outcomes with flipped
classrooms** - *a collection of
empirical evidences*

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Flipped Learning is a New Fad in Education

will it last???



1. Learn Independently
(before class)

videos, books,
web resources



4. Written/Online Homework
(after class)

WebAssign/textbook problems



2. Benchmark Understanding
(before class)

online quizzes (e.g., Learning
Management System)



3. Active Learning

(in class)

group quizzes
due at the end of class

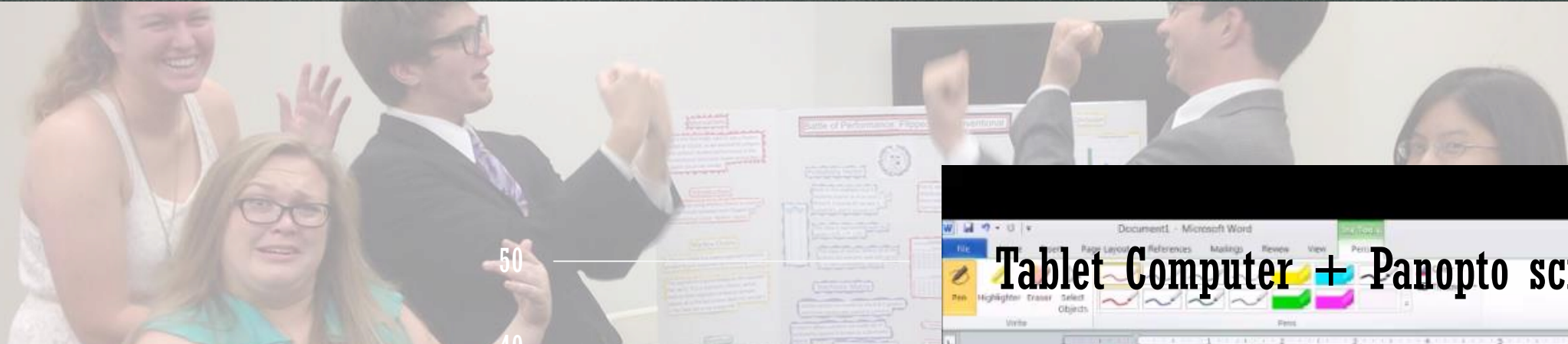
Institutional Background

what works here might not work elsewhere ...

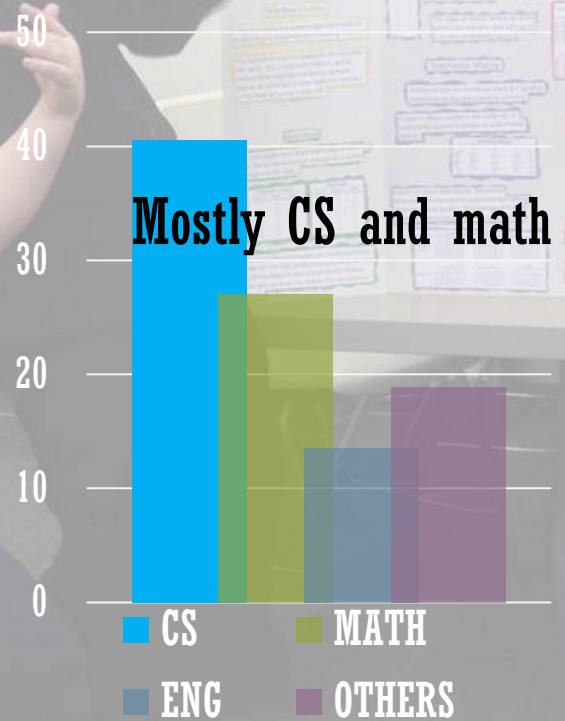
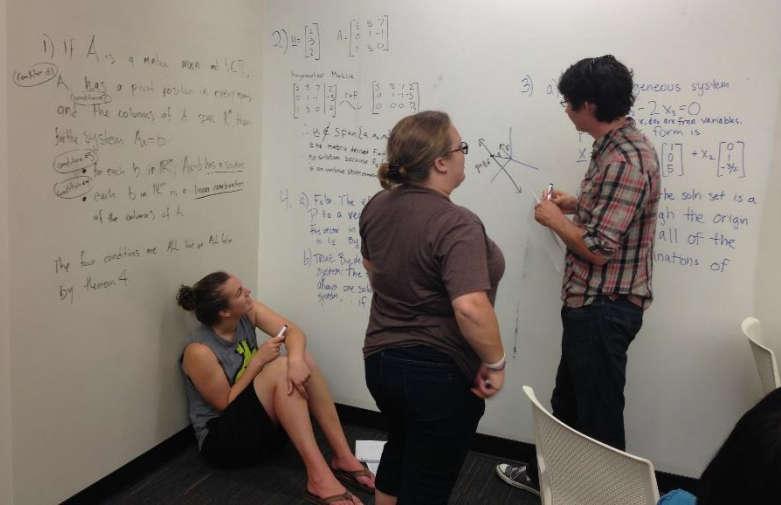
- ❖ Part of the 23-campus California State University System
- ❖ Large urban and comprehensive campus with ~ 37,500 students (Fall' 15 data)
- ❖ ~90% commuters
- ❖ Nearly 2/3 qualifies for financial aid
- ❖ A Hispanic-Serving Institute

(Guinea Pig) Class #1

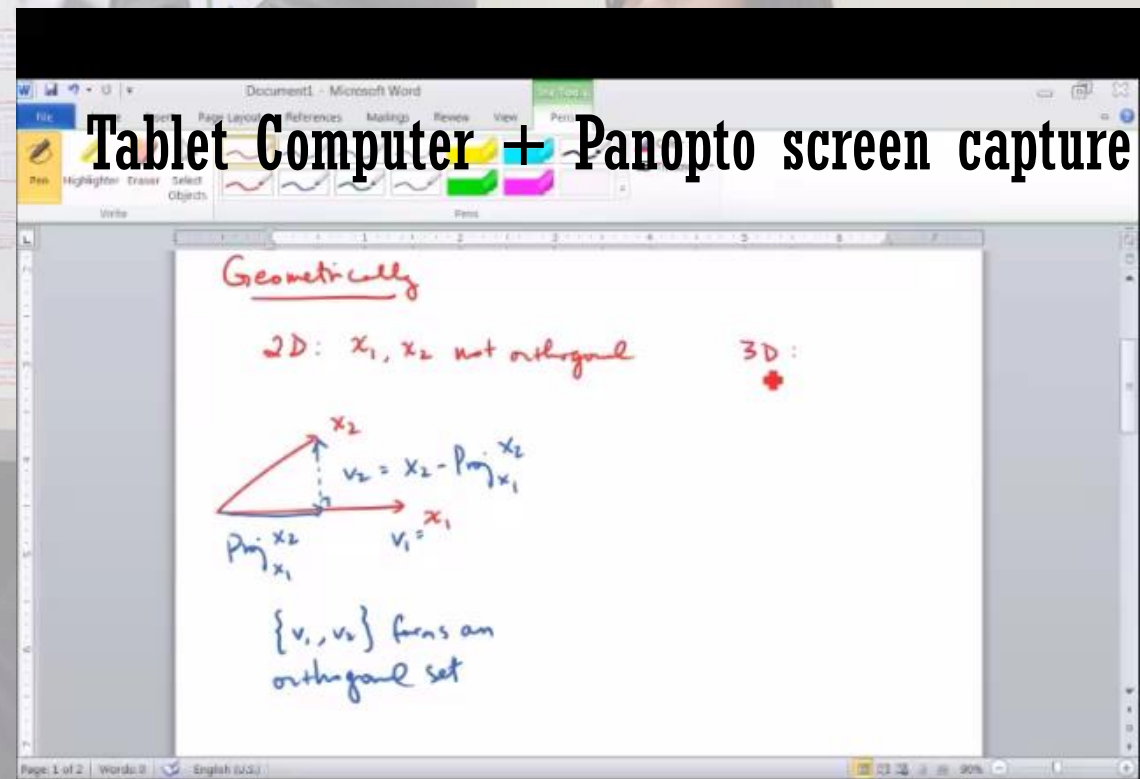
MATH 247: Introduction to Linear Algebra



Active learning classroom



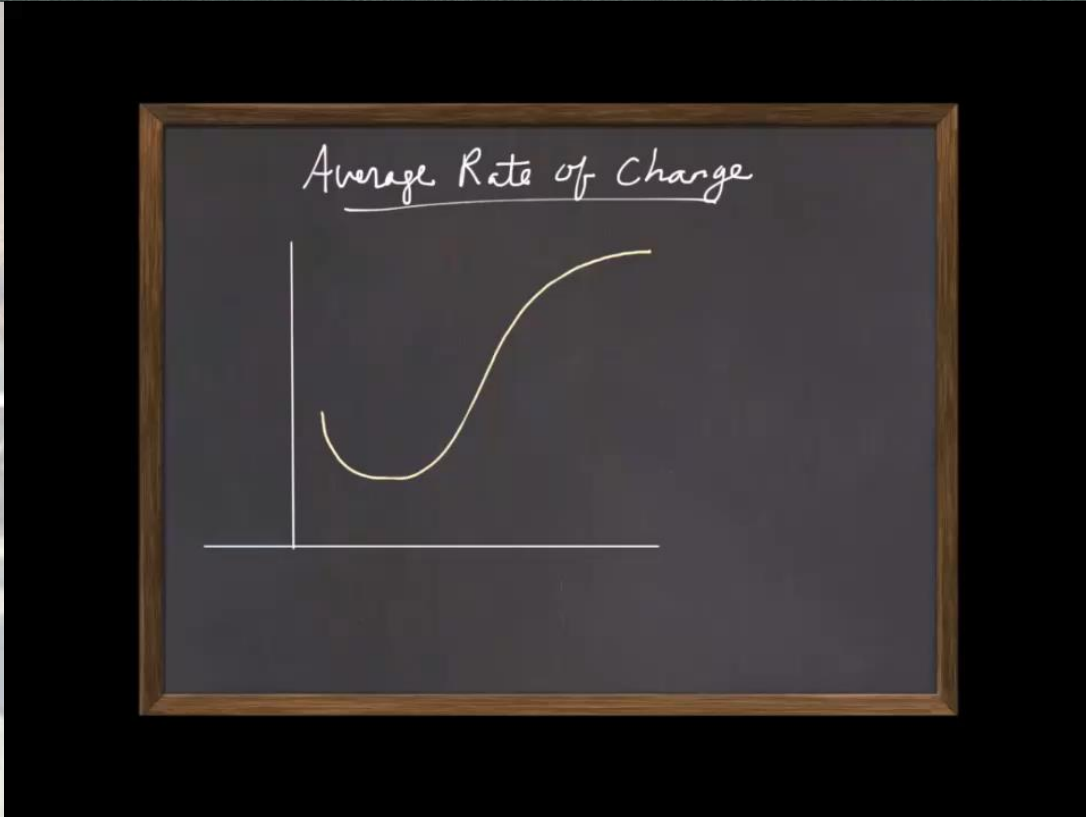
Mostly CS and math



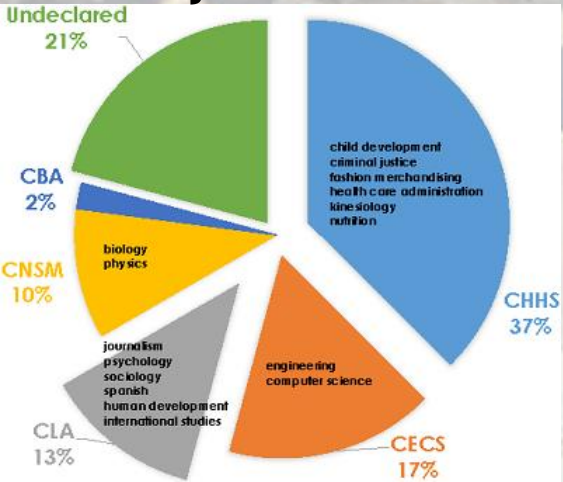
Tablet Computer + Panopto screen capture

(Guinea Pig) Class #2

MATH 113: Pre-Calculus Algebra



Non-majors



iPad + Doceri screen capture + YouTube

Why?

why torture myself with more work (& more  from students) ?

I was mainly not satisfied with math students' abilities to

1. Communicate
2. Write
3. Teamwork

I was even more disturbed by the **ineffectiveness of timed "exams"** – are exams, even when written with extreme care, truly measuring students' faithful understanding of the content?

Food for thought:

Q: Construct a matrix A , not equal to the identity matrix, such that $Ax = b$ is consistent for all b . Be sure to justify why your example works.

2.
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{rrref}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Student response #1

① let $A = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 6 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $\det(A) = 5 \cdot 6 \cdot 1 = 30 \neq 0$
since A is a triangular matrix.
 $\therefore A$ is invertible and through the Invertible Matrix Theorem,
 A is onto and 1-1 which is consistent for all b in $Ax = b$.

Student response #2

Desired Student Learning Outcomes (SLOs)

why fix it if it ain't broken?

[SL01.] Improved math verbal & written communication skills — leads to **mastery of content** beyond the basic levels of Bloom's Taxonomy

[SL02.] Increased depth of understanding on key concepts — leads to better **retention** of course materials

[SL03.] Improved attitudes towards mathematics — key to **persistence** through a rigorous STEM curriculum

[SL04.] Improved (interpersonal) skills in collaborative environment — prepares students for real-world **work place**

Observed SLO1

Improved math verbal & written communication skills

Q: Find a basis for each eigenspace of A.

Non-Flipped, highest exam grade

c). $\lambda = 4$

$$\begin{bmatrix} 0 & 0 & 0 \\ -2 & -3 & 0 \\ 5 & 3 & 0 \end{bmatrix} \quad R_3 \Rightarrow R_2 + R_3$$
$$\begin{bmatrix} 0 & 0 & 0 \\ -2 & -3 & 0 \\ 3 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x_1 = 0 \\ x_2 = x_3 \end{matrix}$$

at $\lambda = 4$, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 1$

$$\begin{bmatrix} 3 & 0 & 0 \\ -2 & 0 & 0 \\ 5 & 3 & 3 \end{bmatrix} \quad x_3 = -x_2$$

at $\lambda = 1$, $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Flipped, highest exam grade

c) $E_{\lambda_1} = (A - \lambda_1 I)x = 0 = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ 5 & 3 & 0 & 0 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$$E_{\lambda_1} = \left\{ \alpha \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R} \right\} \quad x = \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = x_3 \end{cases}$$

$E_{\lambda_2} = (A - \lambda_2 I)x = 0 = \left[\begin{array}{ccc|c} 3 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 5 & 3 & 3 & 0 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$$E_{\lambda_2} = \left\{ \alpha \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R} \right\} \quad x = \begin{cases} x_1 = 0 \\ x_2 = -x_3 \\ x_3 = x_3 \end{cases}$$

basis for Eigenspace of A: $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ ✓

Observed SLO1

Improved math verbal & written communication skills

Q: Construct a non-standard basis of \mathbb{R}^3 and justify why your construction works.

4. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Works because they are not in
REF do not follow IMT

Non-Flipped, highest exam grade

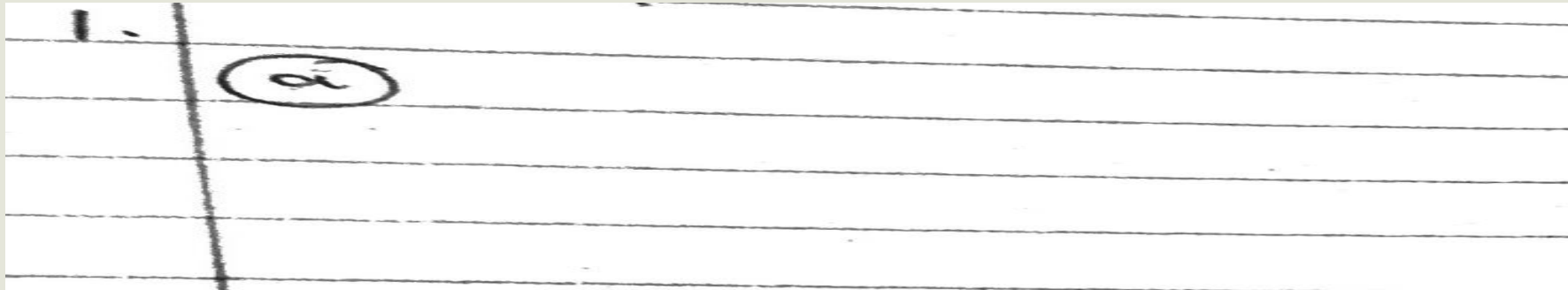
4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{4R_1 + 4R_2} \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a non
standard basis
for \mathbb{R}^3 because
it is a linearly
independent set
that spans \mathbb{R}^3

Flipped, highest grade

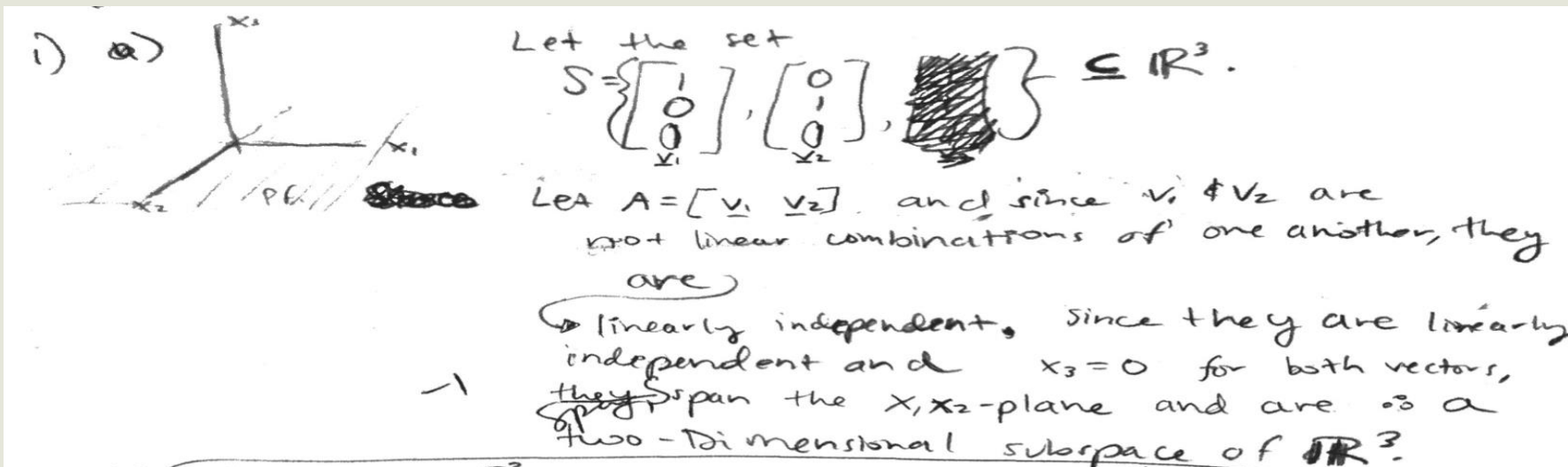
Observed SLO1

Improved math verbal & written communication skills

Q: Construct a non-standard basis of \mathbb{R}^3 and justify why your construction works.



Non-Flipped, lowest exam grade

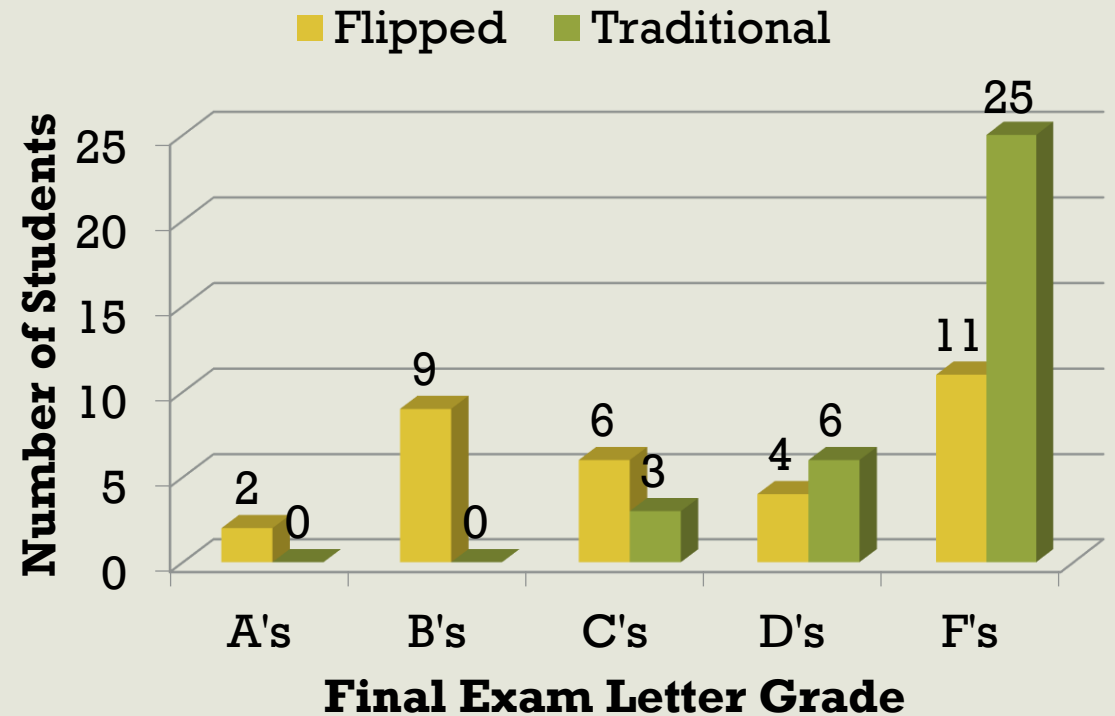


Flipped, lowest exam grade

Observed SLO2

Increased depth of understanding on key concepts [2]

Score (150 possible)	Flipped (n = 32)	Traditional (n = 34)	Percent Difference
Mean	101	88	8.6% ($= (101-88)/150$)
Median	111	79	21.3% ($= (111-79)/150$)
Pass Rate (C or higher)	53.1%	8.8%	44.3% ($= 53.1\% - 8.8\%$)

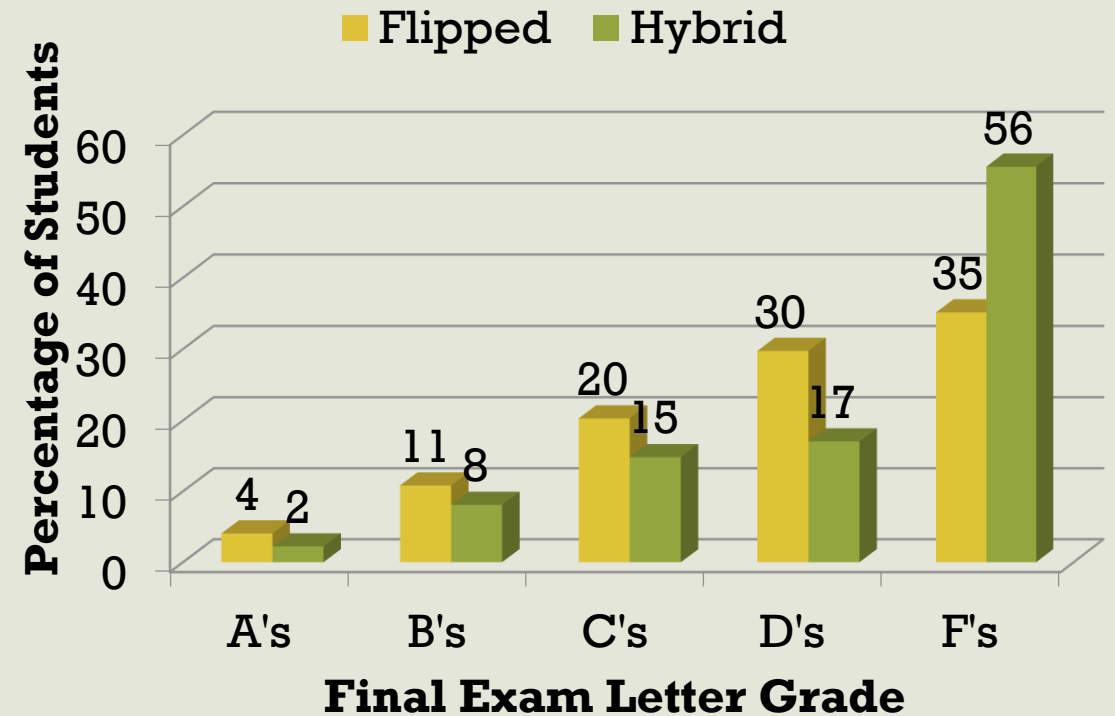


Final exam grade distribution in the introductory Linear Algebra class.

Observed SLO2

Increased depth of understanding on key concepts [1]

Score (100 possible)	Flipped (n = 74)	Hybrid (n = 132)	Percent Difference
Mean	63.3	57.5	5.8% (=63.3%-57.5%)
Median	63.5	56.5	7% (=63.5%-56.5%)
Pass Rate (C or higher)	35.1%	25.8%	9.3% (=35.1%-25.8%)



Final exam grade distribution in the Pre-Calculus Algebra class.

Observed SLO3

Improved attitudes towards mathematics [2]

- ❖ Asked students to agree/disagree with statements on a Likert scale of
1 = Strongly Disagree to 5 = Strongly Agree

- ❖ 45 statements in 4 categories [3, 4]:
 1. **Confidence in learning mathematics** (*e.g.*, “*I think I could handle advanced mathematics.*”)
 2. **Mathematics usefulness** (*e.g.*, “*I will need mathematics for my future work.*”)
 3. **Beliefs about mathematics** (*e.g.*, “*Math problems have one and only one right answer.*”)
 4. **Learning with others** (*e.g.*, “*Talking with others about math problems helps me understand better.*”)

- ❖ Administered to classes at beginning & end of term

Observed SLO3

Improved **confidence** towards mathematics [2]

Positive Statement(s)

Q2: I am sure I could do advanced work in math.

Desirable change with regards to increased confidence in mathematics

Negative Statement(s)

Q8: I don't think I could do advanced mathematics.

Desirable change with regards to decreased negative confidence in mathematics

Q11: Most subject I can handle OK, but I have a knack for messing up in math.

Q12: Math has been my worst subject.

Undesirable change with regards to decreased confidence in mathematics

	Flipped		Traditional	
Median (Mean)	Q2	Q8	Q11	Q12
Post	4(4.18)	2(1.79)	2(2.5)	1(1.8)
Pre	4(3.94)	1.(1.81)	1.5(1.85)	1(1.35)
Paired Sample Diff. in Means	0.2326	-0.1395	0.6824	0.5263
P-value	0.038	0.037	0.011	0.008

[2] Results generated by the Wilcoxon signed rank test

Observed SLO3

Improved **beliefs** towards mathematics [2]

Positive Statement(s)

Q26: There are often several different ways to solve a math problem.

Q27: Time used to investigate why a solution to a math problem works is usually time well spent.

Desirable change with regards to increase in positive beliefs about mathematics

	Flipped		
Median (Mean)	Q26	Q27	Q31
Post	5(4.42)	5(4.39)	2(2.39)
Pre	4(4.25)	5(4.19)	3(2.69)
Paired Sample Diff. in Means	0.093	0.2093	-0.1628
P-value	0.035	0.033	0.013

[2] Results generated by the Wilcoxon signed rank test

Negative Statement(s)

Q31: Math problems have one and only one answer.

Desirable change with regards to decrease in negative beliefs about mathematics

Observed SLO3

*Increased enjoyment **learning with others** [2]*

Positive Statement(s)

Q40: Math is more interesting when I work in a group with other people.

Q38: I prefer to work with other students when doing math assignments or studying for tests.

Desirable change with regards to increased enjoyment learning with others

- 14. My in-class discussions with peers and the instructor help me learn.**
- 15. The class time is structured effectively for my learning.**
- 17. The structure of this flipped class supports my learning in and out of class.**
- 19. Having to communicate mathematics in class help me learn the concepts better.**
- 20. I enjoyed learning in this flipped class.**

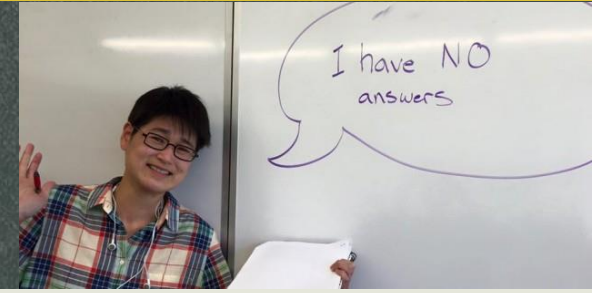
	Flipped	Traditional
Median (Mean)	Q40	Q38
Post	4(3.79)	3(3.27)
Pre	4(3.77)	2.5(3.12)
Paired Sample Diff. in Means	0.2558	0.4737
P-value	0.037	0.43

[2] Results generated by the Wilcoxon signed rank test

	Mean	Median
14.	4.28	4.5
15.	4.03	4
17.	4.13	4
19.	4.34	5
20.	4.13	5

Concluding Remarks

It sounds tempting, but is it worth it?



- When a class is **organized effectively** and **efficiently**, deeper learning can be accomplished through various means of active learning strategies (even in a lecture-based or hybrid class). Strayer [5] noted, and was reaffirmed here, that students can be resistant to the change to their work and study habits brought by the flipped format. Increased predictability and organization can help ease the transition and empower students to find a rhythm that works for them.
- When running a flipped class, there is not a one-size-fits-all bucket list of must-dos. **Do what works for your teaching style** and **your institution**, as suggested by [2, 6].
- Designing a full-blown flipped class requires a lot more preparation time; therefore an interested instructor should be cautious and gauge their **familiarity with the technology**, **mastery of the content**, the **expected student learning outcomes**, and the **program learning outcomes** before embarking on this endeavor.

References

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- [4] Herzig, A and Kung, D. (2003). Cooperative learning in calculus reform: what have we learned? *American Mathematical Society*.
- [5] Strayer, J. (2012). How learning in an inverted classroom influences cooperation, innovation and task orientation. *Learning Environments Research*. 15(2). 171-193.
- [6] Lape, N., Levy, R., Yong, D., Haushalter, K., Eddy, R., and Hankel, N. (2014). *Probing the inverted classroom: a controlled study of teaching learning outcomes in undergraduate engineering and mathematics*. 121st ASEE Annual Conference & Exposition, Paper ID #9475, Indianapolis, IN.