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Self-folding origami membranes

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Abstract – I consider the collapse of a freely-hinged membrane consisting of rigid triangular shapes under the influence of specific, local interactions and a global bending stress. In the absence of the applied stress, the film collapses via a symmetry-breaking process. An external bending stress applied to the collapsing sheet dramatically orients the collapsing domains, giving local control over the mechanical properties of the collapsed sheet, just as an external field would in a magnet.

Introduction. – *Origami*-inspired structures and ideas are increasingly informing the design of state-of-the-art microscopic mechanical devices and molecular assemblies. Mico-electrical-mechanical-systems (MEMS) mirrors have been constructed [1,2] based on a complex layering of flexible and stiff layers to create specific upward and downward folds [3]. Complex three-dimensional structures [4–7] have been induced by taking advantage of a strain-mismatch in a multi-layer causing spontaneous curvature and "rolling-up" of a structure into pinchers, enclosures, hinges, nanoscopic capacitors and other machines. Stress mismatch has been identified, in the guise of a rigid membrane supported by a gently contracting elastic substrate [8] as an explanation for the structure of unfolding botanical leaves [9]. Here, a mechanism controlling the unpacking of an embryonic leaf has been harnessed to engineer a patterned surface, one with a characteristic "herringbone" relief pattern. Here, paper models guided the understanding of the leaf-unfurling process by referring to pattern of folds that had been discovered in the context of efficiently folding automobile maps [10,11]. Thus, the *origami*-design process circulates between folded structures known well to paper-folding artists, towards engineered structures, and finally back to naturally appearing surfaces [12].

Another instance of such a design convergence involves the spontaneous patterns of stress in compressed elastic membranes. These patterns appear in crumpled sheets [13–15], and in a rather more ordered pattern in the bucking of thin-walled cylinders [16,17]. A "folding pattern" inspired by these sorts of "self-organized" creased sheets is the basis for the construction of a novel arterial stent [18]. A similar effect is behind the discovery of hollow, helical silica tubules, self-folded under specific interior interactions [19]. At a smaller scale, complex, yet flexible molecules have been designed with an *origami*-flavor, capable of flexibly bending in a number of directions [20], and graphite sheets have been coaxed to behave as nanoscopic paper, forming tubule "peapod" structures [21]. In the end, the packing of a membrane into a specific volume, requiring outward and inward folds in the right places, is a problem nature, artists, and engineers have solved in turn, each for their own specific purposes.

While complex geometries and 3D mechanical functionality are routinely achieved at the meso and macroscale (folded "by hand"), there continues to be a lack of articulated detailed folds at the microscopic and smaller scales. One problem standing in the way of *origami* methods being more widely applied at smaller length scales is the lack of a reliable method to coax an ordered, thin membrane into adopting a particular three dimensional shape. What I offer in this paper is a method, relying on geometry, to force a controlled collapse by manipulating self-interactions and gentle external influences. It is my hope that these methods, culled from the *origami* artistic community, will allow the construction of MEMS machines at unprecedented scales with unprecedented functionalities.

In this paper, I look at the collapse of a pre-creased thin membrane. The nature of the creasing is, in this respect, meant to model the actual pattern of polygonal facets of a manufactured sheet, either through scoring and pre-creasing (for macroscopic papers or metal foils), or achieved through lithographic means (for mesoscopic applications) [1–3,18]. The triangular net of freely-bendable joints holding together a network of otherwise rigid polygons has been used as an approximation in the collapse of smoothly-elastic sheets [13,15,22–26]. Through the application of local weakly attractive forces, such a triangulated membrane can adopt many interesting geometric phases, from roughly flat, to a fractal crumple, to a compact, collapsed "flat-folded" state. As is well known, the pattern of scores must obey several key properties to allow this flat-folded state to exist. As each polygonal facet in the folded structure must either have its normal parallel or anti-parallel to the same direction, it is natural to assign Ising-like variables to each cell [26]. Thus, the folded, flattened configuration must be characterized by an *anti-ferromagnetic* state of these spin variables. An "up" domain must be surrounded by "down" domains, or there must be unfolded creases in the system. Thus, the folding pattern must divide the plane into a *two-colorable* map. This is essentially a restating of the Kawasaki Theorem of flat-folding origamis: each vertex in the folded, collapsed state must assign 180° to the "up" segments at the vertex and 180° to the "down" segments arriving at the vertex, and the up/down domains have to alternate [27]. Any target structure that is designed to fold flat must obey minimally these restrictions in the crease-pattern design. The problem of self-intersection during the collapse [28,29] introduces more interesting, long-range constraints that enrich the underlying spin-model, and dramatically complicate the folding sequence and design.

What is needed for such an *origami*-inspired construction of useful three-dimensional structures from folded flat sheets is a robust method of driving a possibly disordered, frustrated process towards a robust, efficient collapse. I will study exactly such a collapse in terms of the crease pattern of ref. [18] in response to weak internal interactions, and then in the presence of an externally-applied curvature of the sheet. This external curvature in an indentation experiment [30] gives a depression with a very similar geometry to the indentations designed in ref. [18], inspired by an architectural application of paper-art and architecture [31]. Additionally, it is well known that an intrinsic curvature (in this case enforced by using a toroidal sheet) guides a randomly crumpled sheet into new phases [25]. The application of an external curvature is indeed the method *origami* artists use to collapse paper sculptures with the same folded structures.

I will describe the dynamic model I employ to investigate the system first, and describe the features of collapse of a "stent-like" scored sheet in the absence of an applied curvature, then demonstrate the dramatic effect an external curvature can have during the collapse, and offer some conclusions and speculations on the further applications of *origami* collapses to MEMS technology.

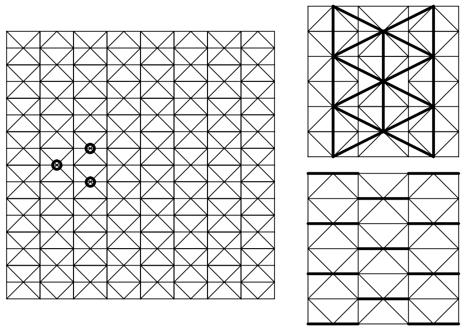

Fig. 1: Crease pattern. On the left, a square sheet with edge length $17D$ has been scored, so that the sheet can flex along any of the lines shown. Three of the "central" vertices controlling the collapse have been indicated by circles. On the right, the two sets of attractive intra-sheet interactions are displayed. Each bold line is a spring with zero rest length.

Model. – As I am less interested in the thermodynamic equilibrium of these membranes than their dynamic collapse from an open state, I have implemented a Brownian motion simulation for the sheet [32,33], rather than attempt to locate the global minimum energy state of the sheet directly [13,15,24] or through a Monte Carlo procedure [22,23,25,26]. As I restrict the discussion to relatively weak forces driving the collapse, the sheet is essentially a two-dimensional version of the freely-jointed chain model of flexible polymers. As in fig. 1 the pattern of connections is composed of triangular domains joined at flexible boundaries. The crease pattern that I am investigating is exactly that used to create the arterial stent [18]. This crease pattern differs, in substantial ways, from the pattern of creases one might expect in the crumpling of an unscored elastic sheet. Elastic ridges form in such a system, and typically four such ridges meet at a singular vertex. In such a spontaneously crumpled system, a substantial energy is stored in the bending of the sheet along these ridges, and the interaction of these ridges gives a decidedly non-Hookeian angular restoring force along the ridge [14,24]. The model I study here is physically different in two important respects. First, there is *no* energy associated with bending the sheet along any of its pre-arranged creases. Thus, every configuration of the sheet which preserves the shapes of each of the triangular facets has *no elastic energy* penalty at all. This can be arranged by sufficiently deep scoring, or if the sheet is a polymer film, suitably sever photolithographic degradation of the sheet along these pre-planned creases. The second important way this model differs from a study of spontaneous crumpling of elastic membranes, is that I have specifically chosen a crease pattern wherein six creases meet at each vertex. It is true that the generic situation for a randomly crumpled sheet has degree four vertices throughout, but as is pointed out in ref. [28], any four-fold vertex is characterized (asymptotically) by a single degree of freedom in the four dihedral angles making up the facets meeting at the vertex. Thus, any sheet composed of rigidly flat vertices with only degree four vertices acts like a machine with a *single internal degree of freedom*. Indeed, the Miura map folding pattern is interesting precisely because it turns the folding and unfolding of the map (or solar panel) from a chaotic mess of misfolded layers into a smooth, single degree of mechanical freedom simple machine. However, it is a general principle of mechanical engineering that *adding additional degrees of freedom* helps a mechanical system avoid mechanical instabilities that, in this case, would hinder the collapse of the origami pattern. Indeed, I show below that the Miura pattern with four-fold vertices collapses just as easily as the "stent" crease pattern when additional degrees of freedom are added.

For any crease pattern under study, then, the triangular domains of the pattern must be kept from changing shape. It is sufficient to maintain the distances between all connected vertices. Thus, the model I study below is a *vertex model*. Each vertex obeys an equation of motion:

$$b\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = \sum_{j-conn.} \mathbf{T}_{ij} + \mathbf{F}_i, \tag{1}$$

Here, $\mathbf{r}_i$ is the vector position of the i-th vertex in the sheet, b is a drag coefficient and the $\mathbf{T}_{ij}$ forces are tensile forces responsible for maintaining the initial distance between vertices connected by a flexible joint:

$$\mathbf{T}_{ij} = -k\hat{\mathbf{r}}_{ij}(r_{ij} - l_{ij}) \text{ when } i \text{ and } j \text{ are connected}, \tag{2}$$

$$\mathbf{T}_{ij} = 0 \text{ otherwise}. \tag{3}$$

Here, r_{ij} is the distance between vertices i and j, and l_{ij} is the distance between the vertices in the *unfolded sheet*, and k is the spring constant for fluctuations of the bond lengths. Thus, the in-sheet distances are allowed to fluctuate harmonically around their given initial lengths. A more complex method for ensuring the constant-length constraints exists [32,33], but as long as all other driving forces are small compared to the elastic restoring force above, the bond-lengths will remain as in the initial sheet. The force $\mathbf{F}_i$ contains all other interactions in the system:

$$\mathbf{F}_i = \xi_i + \text{attractions}, \tag{4}$$

where ξ_i is a Gaussian random noise chosen so that thermally activated extensions and contractions of the bond lengths do not exceed 1% of a bond length during the simulation:

$$\langle \xi_i(t)\xi_k(t')\rangle = 2bk_BTD^2\delta_{ij}\delta(t-t'). \tag{5}$$

Here, D is the underlying lattice scale for the crease-pattern (as in fig. 1), and the effective temperature k_BT is chosen so that

$$|\xi| \approx 0.01kD, \tag{6}$$

thus choosing a dimensionless noise, $|\xi| \approx 0.01$, and dimensionless spring and lattice constants $D = 1, k = 1$, and the unit of simulation time to be b/k.

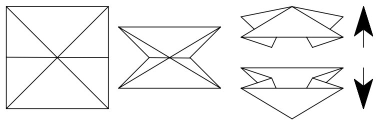

Fig. 2: The collapse of a single domain, either "up" or "down" The "central vertex" is at the center of each square-creased domain.

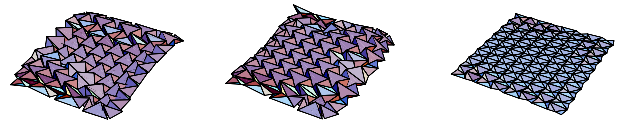

Fig. 3: Typical configurations of the collapsed sheet. Two line defects in the overall "down" configuration are clearly visible in the two left-most configurations.

The attraction term is a short-ranged restoring force shown schematically in fig. 1 above as the bold lines. These lines link various pairs of vertices with elastic attractive forces on the scale of the thermal noise for the flattened sheet. I have modeled two kinds of elastic forces. The first, indicated schematically by the bold lines in the upper-right panel of fig. 1 models the effect of supporting the entire crease pattern with a soft, elastic substrate which, as in ref. [8], is applying an isotropic compressive stress to the membrane. This is admittedly a crude approximation to the underlying continuum mechanics of the underlying soft elastic medium of ref. [8] but my purpose here is to focus on the mechanics of the sheet itself. The second set of interactions impose weak interactions between spots in the sheet directly opposite the intersection of six crease lines. These vertices, when brought together, force each square domain of the crease pattern to collapse as in fig. 2 so that the "central vertex" points either "up" or "down". Thus, the substrate-mediated interaction drives each of the central vertices to point in the *same direction as their neighbors* giving an effective Ising interaction between the domains. On the other hand, the purely compressive interaction (bottom right panel in fig. 1) gives no such bias, so correlations between central vertices are mediated solely by the crease-pattern tensions, $\mathbf{T}_{ij}$. These compressive interactions could be engineered lithographically by decorating the scored sheet with solvent-sensitive electroactive polymer domains.

The equation of motion for each vertex, eq. (1), is numerically integrated over a total simulation time of 30 in dimensionless units. In each case below, two hundred runs starting from a flat sheet have been executed, over independent realizations of the thermal noise.

Collapse. – In fig. 3 I show typical configurations of the collapsed sheet under both types of internal interactions. Large domains of vertices with common alignments are visible.

Interestingly, roughly 40% of the generated configurations were basically uncollapsed, as in the rightmost

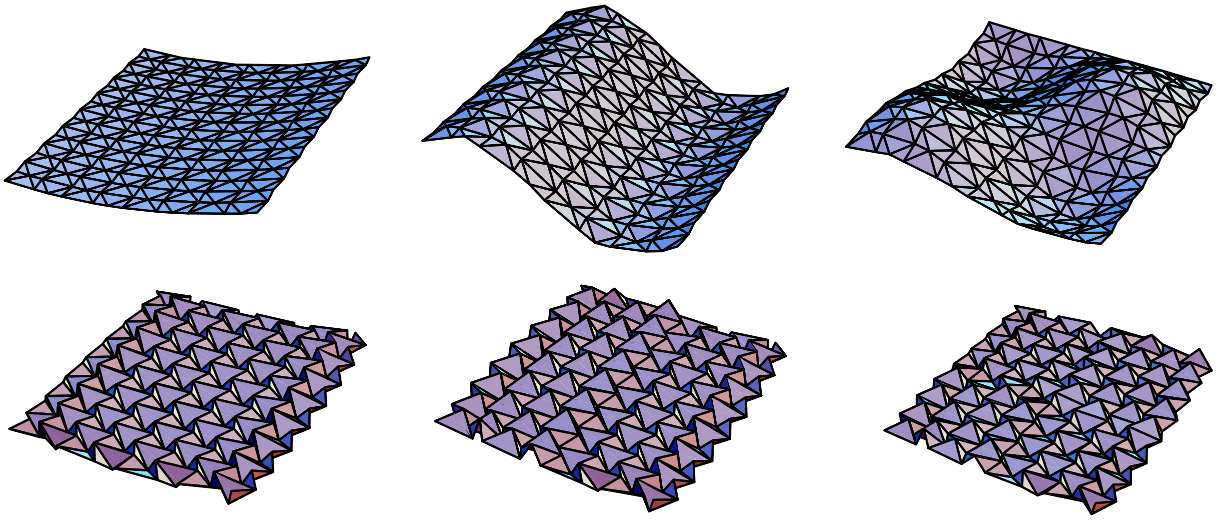

Fig. 4: Initial (top row) and final (bottom row) configurations of the sheet subject to a bent initial state. The initial profile of the sheet has been exaggerated in this plot by a factor of 10. Note that the ordering of entire regions of vertices as "up" or "down" follows the initial curvature of the sheet faithfully.

configuration in fig. 3. As the interactions themselves are up/down symmetric, and the crease pattern itself has no intrinsic tendency favoring the central vertices to point either up nor down, we have a spontaneously broken symmetry. Thus, even a very weak bias in the initial collapse will lead to remarkably ordered behavior of the collapsed membrane. Thus, we have a system that manifestly has a multitude of ground-state configurations (each of the central vertices has a discrete degree of freedom in the collapsed structure), and the disordered, flat state has a relatively long life. In such a case, a quenched external field can serve to break the initial symmetry of the problem, and provide a sure guide to collapsing the membrane in an *a priori*, designed state.

This is indeed the case. The biasing field in this case is geometric. In fig. 4, I show the main results of this study. In each of the three cases displayed, the initial state of the sheet has been biased by bending the sheet in a particular manner. The upper row of figures show the sheets in their initial configurations. The left figure has the sheet bent upwards in a parabolic arc, the middle panel has a sinusoidal corrugatated profile, and the right panels have their initial state as

$$z_{ij} = \epsilon \sin(2\pi i/L) \sin(2\pi j/L) \tag{7}$$

where i, j index the position of each vertex, and L is the side-length of the sheet (in this case $L = 17$). In each case, the maximum deviation of initial sheet from absolutely flat is small ($\epsilon = 1$ for $L = 17$ in this study). These final configurations are repeatable over *200 different realizations of the thermal noise*. In *each case* the central vertices point *opposite* to the imposed initial local curvature of the sheet. The small initial bias guides the self-folding of the sheet towards a unique final state among the myriad of equivalent energy ground states of the system. Thus, the *linear response* of the system to an initial condition is enough to guide the entire future behavior of the system. This property of collapsing *origami* sheets is well known to *origami* artists. A combination of gentle bending and crosswise compression is often sufficient to gain the collapse of a structure in a desired state. Here, I have taken that paper-artist strategy and validated it through these small-scale simulations. Indeed, the buckle pattern for an axially compressed thin-walled cylinder [17] shows the same tendency for inward dimples to appear in an ordered manner. The specific prediction here is that, had the stiff overlayer in ref. [8] been pre-creased in the manner of fig. 1, and had the substrate been curved likewise in the manner presented in fig. 4, then as the underlying soft elastic substrate dried, *ordered* surface patterns such as those appearing in fig. 4 would have appeared.

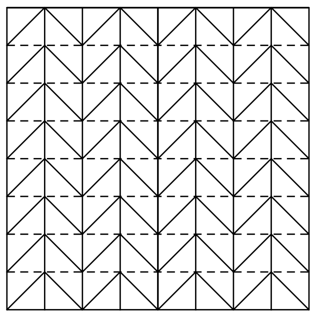

Fig. 5: The Miura crease pattern investigated, with (extra) "unfolded" crease lines (dashed).

Miura sheet. – However, quite a different pattern appeared spontaneously in the drying experiment of ref. [8]. Indeed, the crease pattern that appeared

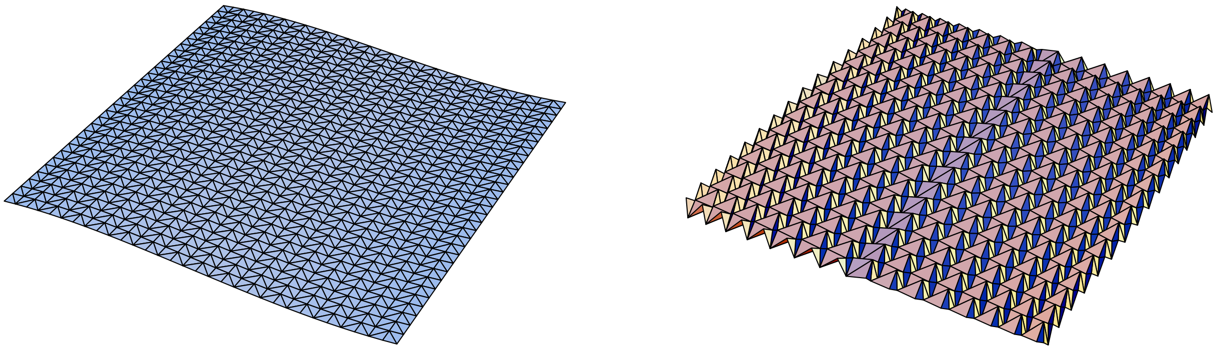

Fig. 6: A Miura map-fold crease pattern subjected to an initial sinusoid curvature (on the left) converges to a ordered Miura patterns separated by a line defect (right panel). The addition of "unfolded" creases (dashed lines in fig. 5) avoids jamming of the pattern during collapse.

spontaneously in that experiment matches that presented here in fig. 5. The pattern that emerged spontaneously from a *flat*, *unscored* overlayer was characterized by a disordered Miura pattern, characterized by a high density of defects in the pattern. This is hardly surprising, given that the Miura pattern when folded by hand has the property that it is identical when viewed from above or below. Thus, the collapsing sheet has no information *in its emergent crease pattern* about weather any particular crease should curve away or toward the substrate. This "isoarea" symmetry essentially frustrates the incipient crease pattern, with the result that a highly defected collapsed pattern emerges.

Large areas of controlled Miura-like collapse have been established through the application of an initial curvature of the sheet, as in fig. 6. I have added additional degrees of freedom which are not expressed in the final collapsed state, but which give additional dynamic freedom for the sheet to arrive at the global minimum energy state I have selected. In this case, a line defect has been induced, and a single line defect at that, down the center of the sheet. It should be stressed that, without bending of the sheet, this defect line is impossible to achieve.

Conclusion. – In this paper, I have demonstrated via a Brownian dynamics simulation, overall control of the final geometry of a regularly creased stiff membrane. A myriad of collapsed states occur when the membrane's collapse is guided by internal, symmetric interactions, but a small initial curvature is sufficient to guide the system to a unique, pre-determined state. Having them all point in the *same* direction is a prerequisite for making a cylindrical arterial stent [18] with its surprising negative Poisson's ratio. Thus, these collapsed sheets have interesting mechanical properties, giving a new way to design the transmission of forces in a microscopic machine. Indeed, I have looked at a very simple sheet here. More mechanical functionality can be built into similar sheets by complicating the initial set of creases. Thus, *origami*-inspired MEMS devices, self-folded in industrial quantity with controllable quality seems a realistic goal.

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