1. Consider the following information for a consumer of pizza and salad:

Indifference Curve 1 includes the following points:

<table>
<thead>
<tr>
<th>Pizza</th>
<th>Salad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

Indifference Curve 2 includes the following points:

<table>
<thead>
<tr>
<th>Pizza</th>
<th>Salad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Price of Pizza = 8
Price of Salad = 4
Income = 48

a) Draw indifference curve 1 and the budget constraint for this individual.

b) At what point [or range of points] is the maximum utility attained [Note: for the purposes of this problem, assume that the indifference curve is a series of straight line segments and not a continuous curve – choose the mid-point of the line segment]

The tangency occurs between the points ((4,4) and (6,3) on both the indifference curve and the budget constraint. If we choose the mid-point, we are at (5,3.5)
c) What is the marginal rate of substitution at the point where the consumer maximizes utility subject to the budget constraint [Again choose the mid-point]?

The marginal rate of substitution at the point where utility is maximized could be calculated directly (the slope of the indifference curve between the two points is -1/2) or it could be noted that at the optimum point, the MRS is equal to the ratio of the prices, so it is -1/2.

d) What is the change in demand if the price of salad increases to 8 [Again choose the mid-point]?

![Situation After Price Increase](image)

The new utility maximization point occurs along the segment between (2,4) and (3,3), or if we take the mid-point (2.5,3.5)
2. For each of the following, use the budget constraint/indifference curve diagram to show what happens to the demand curve for good X:

   a) A change in preferences that favors good X over good Y.

   With the change in preferences, the indifference curves increase moving out more in the direction of good X. At each price (and holding income constant), the individual will demand more of good X and less of good Y. This is a shift out of the demand curve for good X.

   b) An increase in the price of a substitute for good X when good X is a normal good

   With gross substitutes, the increase in the price of good Y leads to an increase in the demand for good X (holding the price of Good X fixed). The substitution effect is positive for Good X and the income effect is negative (if X is a normal good). The overall effect is positive if the substitution effect is larger than the income effect. This is a shift out in the demand curve.
c) An increase in the price of a complement for good X

When the price of a complement increases, demand for good X decreases. This is a shift to the left in the demand curve.
d) An increase in income when X is a Giffen good

If X is a Giffen good, it is an inferior good and the income effect is larger than the substitution effect. This means that the good has an upward sloping demand curve. When income goes up, the quantity demanded of good X goes down. This is a shift to the left of the demand curve.

3. Example with Calculus

Please consider an individual that consumes two goods (X and Y) and has a Cobb-Douglass Utility Function of the form:

\[ U = 4 X^4 Y^6 \]

The individual has income of 100, the price of Good X is 10 and the Price of Good Y is 20.

a) Find the Marginal Rate of Substitution as a function of the quantities consumed of Good X and Good Y [note: you should not use the budget constraint for this]

\[
\text{MRS} = \frac{\alpha A X^{(\alpha - 1)} Y^{(1-\alpha)}}{Y / X} = \frac{\alpha A X^{(\alpha)} Y^{(1-\alpha)}/X}{X / Y} = \frac{\alpha}{1-\alpha} \frac{Y/X}{Y/X} = \frac{.4}{.6}(Y/X)
\]

b) Write out the Lagrangian for this problem
The Lagrangian is:

Maximize \( \mathcal{L} = 4X^4Y^6 + \lambda (100 - 10X - 20Y) \)

\( X, Y, \lambda \)

c) Solve to find the demand for Good X, the demand for Good Y, and the highest level of utility for this individual

To Solve: Differentiate with respect to X, Y and \( \lambda \), then solve:

\[
\frac{\partial \mathcal{L}}{\partial X} = 4 \cdot 4 X^{-0.4} Y^{0.6} - \lambda \cdot 10 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial Y} = 4 \cdot 6 X^{0.4} Y^{-0.4} - \lambda \cdot 20 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = 100 - 10X - 20Y = 0
\]

The easy way to solve is to rearrange the top two equations, then divide one by the other to get:

\[
(0.4/0.6) \frac{Y}{X} = 10/20
\]

Note that \( Y = 5 - 0.5X \) from the budget constraint and substitute:

\[
(0.4/0.6) (5 - 0.5X)/X = 10/20
\]

\[
(0.4/0.6) (5) = 0.5X + (0.4/0.6) \cdot 0.5X
\]

\[
(10/3) = 5/6X
\]

\( X = 4 \)

Then \( Y = 3 \)

The Utility attained when \( X = 4 \) and \( Y = 3 \) is

\( U = 4 \cdot 4^4 \cdot 3^6 = 13.46 \)

d) What are the demand for Good X, the demand for Good Y, and the highest utility level for this individual if the price of Good Y changes to 40?

One strategy would be to go through the same process as part c), replacing 40 for the price of good Y. Also note that:

\[
X = \alpha I/P_x = (0.4) \frac{100}{10} = 4
\]

\[
Y = (1-\alpha) I/P_y = (0.6) \frac{100}{40} = 1.5
\]

\( U = 4 \cdot 4^4 \cdot (1.5)^6 = 8.883 \)