(1, 3 pts ea) **ROULETTE** is a casino game where a numbered wheel spins and a steel ball falls into a location marked by one particular colored number. In the United States there are 18 locations colored red, 18 locations colored black and 2 locations colored green. The red and black locations are numbered 1-36 and the green locations are labeled “0” and “00” as shown in the picture to the right. The wheel therefore has 38 locations in total. Note that the odd and even values are not evenly distributed within each color. For any particular wager a player makes, an expected profit can be calculated from:

\[
\text{Expected profit} = (\text{Prob. of winning}) \times (\text{winning payout amount}) - \text{wager}
\]

(a) One potential wager is to "bet on black" where you make a wager and if the ball lands in a black location you win an additional amount equal to the wager for a total payout equal to twice the wager. For example, if you bet $1 and win the casino gives you a $2 payout. If the ball lands on red or green you would lose your wager and the payout would be $0.

**What is the expected profit from making a $5 "bet on black"?** (round to closest penny)

\[
\text{Expected profit} = \frac{18}{38} \times 10 - 5 = 0.47368 \times 10 - 5 = 4.7368 - 5 = -0.26 
\]

(b) To "let it ride" is to make a bet and if it wins to make the same bet with the entire payout amount. For example, if you bet $1 on black and win the casino gives you $2, you then "let it ride" by betting that $2 on black and if you win again the casino would give you $4. If the ball were to land on red or green on either spin of the wheel you would lose your wager and the final payout would be $0.

**What is the expected profit for betting $7 on black and if you win, "letting it ride" once?** I.E., betting on black and if you win betting your winnings on black again? (keep in mind that if you lose the first bet you are done; round to closest penny)

The two spins are independent so you can multiply the probabilities.

\[
\text{Expected profit} = \left(\frac{18}{38}\right)^2 \times 28 - 7 = 0.22438 \times 28 - 7 = 6.28255 - 7 = -0.72
\]

(c) Another potential wager is to "bet on odd" which has the same payout as betting on black, but wins only when the ball lands in a location with an odd number of either color. If the ball lands on an even numbered location or a green space you lose your wager and the payout would be $0.

**What is the expected profit from placing two bets on a single spin - $9 on black and $9 on odd?** (round to closest penny)

Hint: you may find it helpful to use a tree diagram.

There are 4 mutually exclusive possible results, we need to calculate product of probability of each and payout then sum these terms and subtract the original $18 wager

- Black and Odd: \( P(\text{black & odd})(\text{payout}) = \left(\frac{18}{38}\right) \times \left(\frac{10}{18}\right) \times 36 = 0.21053 \times 36 = 7.578947 \)
- Black and not Odd: \( P(\text{black & not odd})(\text{payout}) = \left(\frac{18}{38}\right) \times \left(\frac{10}{18}\right) \times 18 = 0.26316 \times 18 = 4.736842 \)
- not Black and Odd: \( P(\text{not black & odd})(\text{payout}) = \left(\frac{20}{38}\right) \times \left(\frac{10}{20}\right) \times 18 = 0.52632 \times 18 = 4.736842 \)
- not Black and not odd: \( P(\text{not black & not odd})(\text{payout}) = \left(\frac{20}{38}\right) \times \left(\frac{10}{20}\right) \times 0 = 0 \)

\[
\text{Expected profit} = 7.578947 + 4.736842 + 4.736842 + 0 - 18 = 17.05263 - 18 = -0.95 
\]

See figure on next page for a helpful tree diagram of where these probabilities come from.

(d) In Europe Roulette is more popular than in the US. European wheels do not have a "00" location, but the payout amounts for betting on black, red, odd, even, etc. are the same. Provide an explanation for the increased popularity of roulette in Europe based upon the comparison of an example expected profit calculation. **Be clear and precise.**

Fewer green spaces increase the odds of winning for the players, meaning they lose money more slowly. For example, if you bet $1 on black in the US your expected profit is \(\frac{18}{38}\times 2 - 1 = -0.053\) but if you bet $1 on black in Europe your expected profit is \(\frac{18}{37}\times 2 - 1 = -0.027\) which is not as bad. You can play longer with the same starting amount before losing it ... on average.
The following data comes from the National Cancer Institute (www.cancer.gov). The number of cancer cases estimated for 2008 for the 12 most common cancers (not including non-melanoma skin cancer) are given here:

<table>
<thead>
<tr>
<th>Cancer Type</th>
<th>Estimated New Cases</th>
<th>Estimated Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bladder</td>
<td>68,810</td>
<td>14,100</td>
</tr>
<tr>
<td>Breast</td>
<td>184,450</td>
<td>40,930</td>
</tr>
<tr>
<td>Colon and Rectal</td>
<td>148,810</td>
<td>49,960</td>
</tr>
<tr>
<td>Endometrial</td>
<td>40,100</td>
<td>7,470</td>
</tr>
<tr>
<td>Kidney (Renal Cell) Cancer</td>
<td>46,232</td>
<td>11,059</td>
</tr>
<tr>
<td>Leukemia (All)</td>
<td>44,270</td>
<td>21,710</td>
</tr>
<tr>
<td>Lung (Including Bronchus)</td>
<td>215,020</td>
<td>161,840</td>
</tr>
<tr>
<td>Melanoma</td>
<td>62,480</td>
<td>8,420</td>
</tr>
<tr>
<td>Non-Hodgkin Lymphoma</td>
<td>66,120</td>
<td>19,160</td>
</tr>
<tr>
<td>Pancreatic</td>
<td>37,680</td>
<td>34,290</td>
</tr>
<tr>
<td>Prostate</td>
<td>186,320</td>
<td>28,660</td>
</tr>
<tr>
<td>Thyroid</td>
<td>37,340</td>
<td>1,590</td>
</tr>
</tbody>
</table>

(a) What is the overall probability that a person that has any one of these cancers will die? (round to nearest 0.001)

\[
\text{prob} = \frac{399,189}{1,137,632} = 0.351
\]

(b) What is the probability that a person that has leukemia will die? (round to nearest 0.001)

\[
\text{prob} = \frac{21,710}{44,270} = 0.490399
\]

(c) Is the probability of dying if you are diagnosed with leukemia independent of the probability of dying from cancer if diagnosed with cancer? (demonstrate why or why not with a mathematical calculation)

No, because \( P(\text{die of cancer if cancer} \mid \text{leukemia}) \neq P(\text{die of cancer}) \), 0.490 \neq 0.351

(d) What does your answer to (c) mean in plain non-technical language?

The risk of dying from cancer depends on whether you have leukemia or a different type of cancer.