Hi. This is Professor Jen-Mei Chang with California State University, Long Beach. In this video lesson today we'll look at slant asymptotes for rational functions. Rational function whose degree of the numerator is greater than the degree of the nominator function is likely to have a slant asymptote. In such case we use long-division algorithm to express the rational function in the form of R of X equals the quotient, Q of X, plus remainder R of X, divided by the divisor polynomial, D of X. Division algorithm guarantees that the degree of the remainder polynomial is always less than the degree of the divisor polynomial, which means that as X goes to infinity this expression would eventually go to 0 because the denominator polynomial dominates over the remainder polynomial on top, which means over time as X grows larger this expression gets to be smaller.

Eventually goes to 0. What this means is as X goes to positive or negative infinity the rational function R of X approaches the quotient function, Q of X. This Q of X function is called the slant asymptote. It's also known as the oblique asymptote. And the reason why it's called a slant asymptote is because it's neither a horizontal line or a vertical line. It's given by some function. This Q of X could be any function. It could be linear polynomial, it could be a quadratic polynomial, it could be even higher degree polynomial. We don't know. That's why it's typically a curve instead of a straight line, horizontal, or a vertical line. Let's look at an example next. In this example let's sketch the function R of X who's numerator polynomial is degree 2 and the denominator polynomial is of a degree 1. It satisfies the condition that a degree of a numerator is greater than the degree of the denominator.

First step of sketching a rational function is always to factor the numerator and the denominator. In this case, the denominator is already in its reduced, most factored form. So we just have to worry about the top. We have factors 1 and negative 5, which gives us X plus 1 times X minus 5 or divided by the X minus 3. Next we find the X and a Y intercepts. The X intercepts we set the rational function to 0. And you'll recall that the rational function can only be quoted 0 if the numerator function is equal to 0, which means we're going to set this top portion equal to 0 and that means X has to be negative 1 or 5. For the Y intercept, that's when we set the X value equal to 0. So if we go back to the original rational function R of X, here, you simply just replace all X with 0 then we're left with the constant coefficient, which is negative 5 over negative 3, which reduces down to positive 5 over 3.

Next we find the vertical asymptotes and the behavior near the asymptotes to find the vertical asymptote by setting the denominator function equal to 0. In this case that's X equals 3. To figure out the behavior around the vertical asymptote we pick test points to the left and to the right of the vertical asymptotes. In this case, I'm going to pick 2.9 and 3.1. It's easier to use the factor form of the rational function when we're checking for the signs on the test points. When I use test point of 2.9 back in the original rational function I will get a plus sign for the first factor and a minus sign for the second factor over the minus sign on the denominator, which gives me an overall of a plus sign. This
means that the Y value goes to the positive infinity as X value approaches 3 from the left, which means that near the asymptote the function should have such behavior. For the test point 3.1 I will get a plus first factor, a minus on the second factor, or a plus on the bottom factor, overall gives me a negative sign, which means that the Y value goes to the negative infinity as X approaches 3 from the right. This then gives me a behavior of going downward as I approach 3 from the right.

In step four we find the horizontal asymptote but notice that the degree of the numerator is greater than the degree of the denominator and that is why there is no horizontal asymptote. And this is also why we actually have a slant asymptote instead. In the next step we’re going to find the slant asymptote through the division process. To do the division using long-division with the polynomials or the synthetic division, in this case, because we actually have a factor of X minus 3 on the bottom, you can use the number C equals 3 in your synthetic division process. Here I’m going to illustrate with the long-division. My first term in the quotient would be X to match up with the X-squared in a dividend.

And then I have minus 3X as my second term. When I subtract those two things I have a minus 1X left. I need to carry one term down to match the number of terms in my divisor, and then I need a minus 1 on top to match up with the first term of the new dividend and that gives me a plus 3. It's the constant term. When I subtract those two things I get minus 5, minus 3, and this gives me a remainder of negative 8. That means I can write my original R over X function as X minus 1, as the quotient, plus the remainder, that's negative 8, over the divisor X minus 3. And remember, this quotient function is the slant asymptote. Finally we’re ready to sketch. You might need to plug additional points just to make sure your graph is smooth. As you lay down your axes my first action is always to identify the location of the vertical asymptotes and horizontal asymptotes. In this case it would be the slant asymptote.

A slant asymptote is Y equals X minus 1, which means that it has a Y intercept at negative 1 with a slope of 1. So to do that I'm going to start with negative 1. And the sloping 1, that means I can go up one unit in the right one unit to identify my next point. This gives me two points so then I can connect those two points and make a straight line out of it. Next I’m going to remind myself of the behavior around the vertical asymptote. The function is going to go to the positive infinity as I approach 3 from the left, and the function's going to go to the negative infinity as I approach 3 from the right. Next I’m going to label all the points that the function crosses through. For example, the X intercepts R at negative 1 and 5.

So that means the function needs to go through this point 5. It also needs to go through the point negative 1. The function has Y intercept at 5 over 3. Well that's about 1.67. I think I'm about ready to sketch this graph. I'm going to start from the right-hand side here. I need to draw a curve that gets really close to the slant asymptote and across the point 5 and eventually goes down like that. On the left-hand side I need to have a curve
that gets really close to the slant asymptote as I go up, crosses through the point negative 1, and then come up to cross the point 5 over 3, eventually grows along the vertical asymptotes to the positive infinity. This might not be the best, accurate drawing in the world because the scale's a little bit off. That's why it's called a sketch. You could verify with your calculator or any other computer software just to see that if the general behavior is the same.