Hello, is Professor Jen-Mei Chang at California State University, Long Beach. In this video lesson today we'll look at formulas of some well-known geometric shapes. In particular, we're going to look at three aspects of these shapes. The first one is going to be area, or surface area, depending on the geometry.

The second one is going to be parameters, or it's called the circumference for circles, and the volume. Notice that we can only speak about volumes for objects that enclose a portion, the three-dimensional space. We're going to separate the objects into two different categories. One is called the planar objects. The other one is called non-planar objects. The planar objects are the ones that actually lay flat on the two-dimensional space.

Example, rectangles and circles are your typical planar objects. On the other hand, things that actually have volumes would be nonplanar objects, the cylinders, cones, spheres, and box. These are all nonplanar objects. When you look at rectangle first in the planar objects.

Notice the square is just a special case of a rectangle with all four sides equal. Okay, so let's start with the shape of a rectangle here. The side that is longer is labeled L for length, and the side that is shorter is labeled W for width. So in this picture here, I'm going to label a W here and W there and L top and the bottom here, and notice that rectangle has this feature that has two sets of equal lengths.
You can certainly label you rectangle with any other variables that you're comfortable with. A lot of the times, you can use X and Y and X could be the width or could be the length, Y would be the other one. It doesn't really matter as long as you're comfortable using those symbols. Let's use W and L here, and in terms of symbol, I'm going to denote area by A, and it's going to be length, which is L, times the width, W.

With this definition, area denotes entire amount of region that's enclosed by a rectangle of length, L, and width, W. Another thing we can actually measure for the shape is the overall surroundings. So if you actually count or measure the amount that you get by surrounding all four sides like this. That is defined as the parameter. So that the definition of parameter really is just the sum of all four sides. Using the notation we have here, this would be an L plus an L, plus a W plus a W, which typically we write it as 2 times the quantity L plus W or occasionally you can distribute it in and write it as 2L plus 2W.

It's up to you and also up to the situation which one is easier. In this shape right here, because it's a planar shape, there is no volume we can speak of. Next, let's try to practice translating words into mathematical equations using the definitions we've just learned.

If I tell you that the parameter of the rectangle is 140 feet, and I want to express the area of the rectangle using that information, then what would you write? If we use the notation that we have from before with L and W, then this first piece of information, which tells you that the parameter is 140 feet, allows you to translate that into mathematical symbols of 2L
plus 2W equals 140. Okay, but if you actually express the 2L plus 2W as two times the quantity L plus W, then you can see that we can divide out by 2 both sides to get L plus W equals 70. L could be anything. It could be 1. It could be 2. It could be 3, and then W actually changes accordingly as soon as we specify what L is.

So, for example, if L is 1, then W has to be 69. If L is 2, W has to be 68, and so on. Now we can go back and try to answer the question about the area. By the definition, the area of a rectangle is length times width, which is L times W. Still, this is an expression that involves two unknowns. L and W are both unknowns here, but we do have a constraint. L and W can't just be anything we want.

It has to satisfy this relationship, and this relationship is oftentimes known as the constraint. For example, if you want to get expression that in terms of L only, then we can solve for W in this expression. So if you actually solve for W by subtracting L, both sides, we get 70 minus L for the expression of W. Now that we have an expression for W in terms of L, we can plug that back in to the equation for area, rewrite this in terms of L alone, because now the expression says it's L times W, I have L as in L, but W now is written in terms of L as 70 minus L.

if you actually simplify this expression, you get 70 L minus L squared. So you notice now you've had an area in terms of a single variable of L, which is much easier to manipulate in going for by studying its shape. If, on the other hand, you don't want to express your expression in terms of L, but you want to express it in terms of W, we just do exactly the same thing, but
instead of solving for \( W \), we're going to solve for \( L \). Here from the constraint equation, I can solve for \( L \) by saying \( L \) is equal to 70 minus \( W \), and then once I have that information, I can rewrite my area expression, which is \( L \) times \( W \), but now \( L \) is 70 minus \( W \), and \( W \) is still \( W \).

The question typically won't specify which variable you should express your area in. So I'm just going to use one of them. In this case, I'm going to use the \( W \) expression. Many of the word problems are going to ask you what the length or the width should be, in order to minimize or maximize a certain quantity. So here, you know, in a real-world situation, you want to maximize area as much as you can.

So we're going use the minimum amount of material, which involves the length and the width, to maximize the overall area. The actual question to ask after this step is to find the \( W \) value, so that this expression of area is as large as possible. So we'll do that in a different video, but here we just want to get familiar with manipulating the expressions of parameter and area for rectangular shape. The next planar shape we're going to look at is circle, the radius of \( R \).

Somebody told me that the rounder the circle you can draw freehand, the smarter you are. So let me try this. All right, I think that's not bad. Here I'm going to find the center of the circle here and give it a dot, and out anywhere on the boundary is going to be a radius \( R \). Again, because this is a planar region, there are two things we can talk about. One is area and one is the parameter. The parameter for a circle is really called a circumference.
As you may recall, the area for a circle of radius, $R$ times $R$ squared, circumference of the circle is going to be $2 \pi R$. Now interested in knowing the area or the circumference of a specific portion on the circle. For example, you're trying to figure out how much of a pie or how much of a pizza you actually out of the entire thing, you can actually get that by doing this proportional calculation.

I'm interested in knowing the area of this portion with the angle theta, and what I'm going to do is take the entire area of the circle, which is $\pi R$ squared and multiply it by the proportion that theta occupies in the entire 360 degree circle. Well, theta is how much it occupies, and it's proportional out of the entire thing of $2 \pi$, because the $2 \pi$ gives you the entire 360 degrees. All right, so this theta over $2 \pi$ is the portion that theta occupies out of the entire thing.

So I multiply that in front of the entire area of a $\pi R$ squared. It gives me the area of just that piece of the pie with angle theta. I can cancel the pi out. It leaves me with $1/2 R$ squared theta. So that formula might be familiar to you, as well, but now if you just wanted to find out the arc length, or the portion of the circumference that this angle sustains, then we do exactly the same trick. It's going to be theta over $2 \pi$ is the proportion times the entire $2 \pi R$.

Cancel the $2 \pi$ out. We simply just get $R$ theta. The next shape we're going to look at is the nonplanar shape, and we'll start with a rectangular box here. So let's first try to draw a box. Start with a rectangle. I'm going to tilt it a little bit to get that
perspective with a height of $H$. The trick to this is basically drawing parallel lines everywhere. Another trick to drawing a 3-dimensional object is we use dashed lines for something that is behind that we really can't see. Here we have this side that we really can't see, and this side in the back we really can't see, as well, and this side here we can't see.

This box should have six sides. I'm going to label them using the length, width, and height notation here. For the bottom piece right here, we have $L$ for the longest side. We have $W$ for the shorter side, and height is labeled as an $H$. That's the third dimension coming out of the planar region, and this time, we can't speak of just the area anymore because if I just ask you what is the area of a box, then you're like well which area are you talking about?

Is it the area of the left piece or the right piece? The top piece, bottom piece, or the front or the back, and there's six different sides we can talk about. So simply speaking of area doesn't make sense, because the area really is a word that's reserved for planar objects. Instead, we have to talk about surface area, and in that case, we define the surface area as the sum of each planar area that includes all six sides.

So let's look at area of each of those six sides. Now let's first start with the front side here. The area of this front rectangle comes from the on the $L$ times the $H$, and that's exactly the same as a the back side here. Because the back side also shares the $L$ and the $H$ as its two sides. So the area over there is also $L$ times $H$. The same thing, we can find the area of the left and right side.
So let's look at the right side here, because we've got the symbols labeled on there, and the area over this comes from using the side of $W$ and the side of $H$. So that's going to be $W$ times $H$. The same thing here. It's going to be $W$, $H$, as well, and then we have the bottom and the top. On the bottom side, it's using the side of $L$ and the side of $W$.

This side over here. That means the area over there is going to be $L$ times $W$, and that's going to be the same as the top portion, as well. Now if you add all six sides, it gives you the surface area, because it's really two of the same thing. So then we're just going to pull out the 2, and then have all three other sides, $HW$ plus the $LW$ plus $HL$.

On the other hand, the volume of a rectangular box is defined to be the length times the width times the height, which can be thought of as the bottom area times the height. So in symbols, that's $L$ times $W$ times $H$, and the reason why we can think of it this way, is because it's like stacking up $H$ many rectangles of the area of $L$ times $W$ to give us the overall volume of this thing, and you can visualize it by kind of thinking about it this following way.

Each sheet of this rectangle, which has area of $L$ times $W$ stacked up on top of each other, and if you have each of them all stacked up on top of each other, then you get this overall volume, which is right here. Next nonplanar shape we're going to look at is cylinder with the radius $R$ and height $H$, and it has the shape of the following. So first we start with the kind of the circular top and also a circular bottom. Again, this thing that's in
the back, we can't see. We're going to use a dashed line to indicate that. The height of this thing is H. So I'm going to go down from the center point of the top and all the way to the center point of the bottom.

This distance here is H and the radius out to any point on the circumference is going to be R. This is a nonplanar object. So we can talk about the volume again, and we can talk about the surface area, but they're really three different pieces we have for the surface area. We have the top piece, and we have the bottom piece, and we have the kind of the side piece. We have to be careful when we're reading a word problem is it affects which piece you actually take into consideration.

So, for example if this is a low top and bottom cylinder the in my surface area calculation I to add those pieces in. the area for the top piece would be $\pi R^2$, because that's just a circular shape, and the bottom piece would also be $\pi R^2$, and the question becomes what the area of this middle piece. What we need to do is kind of unfold that piece, and then realize that comes from a rectangular shape.

So pictorially, I can basically cut this piece of paper and then just fold it out into a rectangular piece of paper, and with that, the side edge of this rectangle comes from the edge from here, which is the height H, and the length of this other piece comes from the circumference of the circle, because that's basically unfolding this entire portion of the circle, and then we know the circumference of the circle is $2\pi R$, which means that the area of this rectangle is going to be $2\pi R \times H$, which gives you the area of this middle.
If we're talking about a closed top and bottom, then the surface area, overall, is going to be $2 \pi R^2$, because that's the top and the bottom. That's why the 2, plus the middle piece, which is $2 \pi RH$, but if your cylinder is open top and open bottom, then it simply is just going to be $2 \pi RH$. 