Hello, this is Professor Jamie Chin at California State University, Long Beach. In this video lesson today, we'll look at multiplication and division of rational expressions. To start with the definition, a quotient of two algebraic expressions is called a fractional expression. The keyword here is, fractional. For example, if I write down $2X$ over $X$ minus 1, this would be called a fractional expression because we have two algebraic expressions written in terms of the unknown $X$. Another example would be $Y$ minus 2 divided by $Y$ squared plus 3. Again, both $Y$ minus 2 and $Y$ squared plus 3 are algebraic expressions in terms of the unknown $Y$, and then we write it as a quotient of those two algebraic expressions.

Try to go on with a few more examples yourself to see if you actually understand the definition of a fractional expression. Here I'm going to write down two more, for example $X$ cube minus $X$, divided by square root of $X$ plus 1. Another example would be as simple as just a number on top and something more complicated on the bottom like $X$ squared minus 5$X$ plus 6. All of these four are called a fractional expression. On the other hand, a rational expression is a fractional expression in which both the numerator and the denominator are polynomials. Notice the difference here is, the rational expression, both a numerator and the denominator are polynomial functions. With this definition in mind, which one of those four fractional expressions is not a rational expression? And the answer is, this one, the third one. Because the expression of the square root of $X$ plus 1 is not a polynomial.

But here, we're going to focus on learning techniques to simplify rational expressions where the numerator and the denominator are polynomial expressions. In particular, we're going to look at the multiplication and division techniques. And in general, the reason why the rational expressions are easier to deal with is because we have systematic tools such as factoring to simplify polynomial functions. On the other hand, if your expression that involves square root or cube root or any sort of a power and then it becomes relatively harder to come out with a systematic tool to do that. To start with a very simple rule here, recall that if we have an expression $A$ times $C$ and divided by $B$ times $C$, or ABC or just some sort of an algebraic expression that involves polynomials. Then we can actually simplify it by cancelling out the common factor $C$ to obtain the expression $A$ divided by $B$. The only thing we have to be careful about is that the factor $C$ is not equal to zero. Otherwise, we can safely cancel out the common factor $C$ to reduce it down to $A$ over $B$. For example, if I give you expression $X$ squared minus 1 divided by $X$ squared plus $X$ minus 2, we want to do something similar which means that we have to factor our expressions into a product and then look for common terms within those products. OK, so on the top, I can use the rule of difference of two squares because I noticed it's $X$ squared minus 1 squared. And as we recall the difference of a square's rule is that, if you have something squared minus another thing squared, then we can write it as the first plus the second times the first minus the second.

Here we have $X$ squared minus 1 squared, so I can write it as the first which is $X$ plus the second which is 1 times the first minus the second, and that's called the difference of two squares. On the bottom, we're going to use just a general factoring rule on the quadratic polynomial by looking for factors that are multiplied to negative 2 and adds to positive 1. So my candidate will be 1 and 2 because I know 1 times 2 is going to be 2, but I needed it to be a
negative 2, and I needed to decide where I should put the negative sign, because the factors have to be add up to be positive 1.

I know that I need to put the negative sign with the 1. So this gives me factor X minus 1 times X plus 2. Then we noticed that the factor X minus 1 is common amongst those two, so by the rule, that we've just stated above here, we can cancel out these common factor and reduce it down to X plus 1 divided by X plus 2. And there's nothing else that's uncommon between those two things. Therefore, that will be our final answer to the problem. Now we're ready to look at how to multiply rational expressions. The rule we're going to follow is the following. If we have a fraction A over B times another rational expression C over D, then we simply just going to multiply the top portion, which is A and the C and we multiply the bottom portion which is B and a D together and we write them as a single fraction like that. Plus we'll get an example next. Here we want to multiply two different fractional expressions, and with the first one is X squared plus 2X minus 3 divided by X3 plus 8X plus 16. And the second rational expression is 3X plus 12 divided by X minus 1.

Our first step in simplifying is always to try to break down each expressions into its factored form, because we consider factor as kind of the building block of expressions. You might want to do this separately on the side, if you're not familiar with or not so quick on your factoring yet, but if you're very, very familiar with how to factor quadratic polynomials and linear polynomials, then I'll go ahead and just do it in the problem. Now for the first piece right here, we look for factors as I multiply it to negative 3. And so potential factors will be 1 and 3, we need to decide where to put the negative sign so that when you add those two factors, it would be equal to positive 2. So then you have to put the negative with the 1, that way when you add those two numbers, you would get the positive 2 that's over here. So that gives us the first set of factors from the top portion of the first rational expression, which is going to be X minus 1 times X plus 3. For the second expression down here, in the bottom, we have factors that multiply to 16 and adds up to 8. But there are actually multiple combinations of factors of 16, you can have 1, 16, you can have 2 and 8, you can have 4 and 4. But you need to add those two things so that it adds up to 8. Also that it seems like 4 and 4 will be an obvious choice here. In terms of that's actually is a complete square in this problem anyway. So this actually are factored down to X plus 4 and X plus 4. For the third expression over here in the second fractional expression, we have 3X plus 12. It is a linear term, which--so we can really use the technique we've just done there, but you can look for simply just the common factor in between two terms. This is your first term, and this your second term. What is actually in common between those two things? It is the number 3, because I noticed that 12 is 3 times 4. So both of these terms have a common factor of 3 so I can take that out and then left over from the first term would be X and the left over from the second term would be this 4.

And it's connected by the additional sign in between. On the bottom right here, it's already in its complete factor form, simply just X minus 1. So I'm just going to rewrite, for the second expression over here, we have 3 times X plus 4 divided by X minus 1. With the rules that we've just stated there, to multiply two fractions like that, we simply just going to put all the top portions together and all the bottom portions together. So we're going to have 3 times X minus
1 and X plus 3 and X plus 4 on top. And notice that, always write the constant number first. And not just because it's for readability, it makes it easier to read. There is no number on the bottom, so we don't have to worry about that. And then the order in which you write these linear factors, it's entirely up to you. I'd like to start with the smallest number, X minus 1 and then X plus 4, X plus 4.

Our next task is then to look for common terms like what we did in the previous question and to cancel. So what's in common here? The factor X minus 1 appears in both top and bottom, so that the factor X plus 4 appears both in the top and to bottom so I can cancel that out. And there's nothing else that's in common between the top and the bottom, so I have left over 3 times X plus 3 on top, divided by the left over X plus 4 on the bottom. And this is in the complete simplified form and that's how I'm going to leave my answer. To divide rational expressions, we're going to follow the rule here. First, a rational expression A over B, and I want to divide that by C over D, then what I'm going to do is to turn it into a multiplication problem which we are familiar how to do now, by flipping the second expression. So of course expressions are going to stay exactly the same, and we're going to turn in to a multiplication problem of the inverse of these things, we're going to inverse the positions of C and D. So now D goes on top and C goes on the bottom.

And that's called the multiplication by the reciprocal. We inverted this problem into a multiplication problem then we know how to do that because we've done the problem like that before, right? So that's just going to be A times D on top and the B times C on the bottom. Let's look at the example next. But here, I wanted to take X minus 4 divided by X squared minus 4 divided by the expression X squared minus 3X minus 4 divided by X squared plus 5X plus 6. Now you can start by simplifying each of those expressions into its factored form and then do this in version, multiplication by the reciprocal thing. Or you can do it first, and then simplify from there. It's entirely up to you how you want to do that, depending on your preference. I'm going to do a kind of combination of the two things, I'm going to factor each of those expressions in its original position and then re-write the problem into a multiplication problem.

So I noticed that here for this expression, it's going to be X minus 4 and X minus 1 because, when I take the factor 1 and negative 4, I get a negative 4, as multiply you get negative 4 and you add those two things, you get negative 3. Factors for the X squared plus 5X plus 6, it's going to be 2 and a 3 here, because 2 times 3 is 6 and 2 plus 3 is going to be 5. So I have X plus 2 times X minus 3. And for the first expression here, this is already in a linear factor form, no need to do any work there. This again, a difference of two squares because I noticed that 4 is 2 squared. So this is X squared minus 2 squared and X is the first, 2 is the second, so I'm going to have X plus 2 times X minus 2. Now, every single expression here is already in its completed factored form, then I can turn this into a multiplication problem by multiplying the reciprocal of the second expression.

So just carefully re-write the first portion stays exactly the same, they're now in the completely factored form, so it's X minus 4 divided by X plus 2 times X minus 2. For the second expression, notice that we're going to turn this into a multiplication problem and then inverse the positions at the top and the bottom, so this now, the top portion becomes the denominator,
just $X + 1$ times $X - 4$. Bottom portion of the second expression here, now goes to the top. And then once you've turned it into a multiplication problem, we're just going to simplify it like how we did it in the previous multiplication problem there, straight across and simplify. OK, you can even do those in its position. So $X - 4$ is on top, there is one factor here, and one factor there, in the bottom, $X + 2$ appears in the top and on the bottom.

And that's pretty much it. Nothing else appears both in the bottom and the top. We have just on top, $X + 3$ left, and on the bottom, $X - 2$ times $X + 1$. Notice that if you only have a single factor left, you don't need to use a parenthesis around it, you could but you don't have to, I just--but this is going to be the final answer.