Hello this is Professor Jen-Mei Chang at California State University Long Beach. In this video lesson today we'll look at how to graph and model piecewise functions. I'm going to show you the thought process through examples. Sketch the graph of the following function. The first thing you should notice is that there are two pieces to this graph a the splitting point is defined by this portion of the information. Notice that you're going to use the first piece which is the X square piece for X values that is less than or equal to 1 and you're going to use the second piece that is the 2X plus 1 if X value is greater than one. So a suggestion to you is that once you lay down your corner system you immediately identify where the splitting point is.

In this case, X equals one and then you're going to realize to the left of the splitting point we're going to try to graph the equation Y equals X square because that's the first piece and then to the right of the splitting point we're going to use the second piece which is the Y equals 2X plus 1 and as we're trying to graph those two pieces we're going to use prior knowledges about known functions such as the X, X squared and X cubed or square root or absolute value functions that you have already graphed before. The first thing you do is actually figure out where that end point is located on the coordinate plane or when X is equal to one, I evaluate this function at 1 I will use the first piece which is 1 squared that gives me 1. So this tells you that the point 1, 1, now this one here comes from using the X value 1 and this one here come also from the [inaudible] value after the function evaluation. This means that the graph should go through the point 1, 1 which is right about here on a coordinate plane and to the left of it, it is going to follow the parabola curve. Hopefully you have some general knowledge about what the parabola curve looks like. In particular for this parabola graph it should go through the point 0, 0 and it should also go through the point negative 1, 1 which is right here and it should also go through the point negative 2, 4 which is about right there. So now if you connect those points here it should give you the parabola curve.

I'm going to put the arrow on the left inside of here because it technically stands forever to the left. Add the X value to earn near 1 but not exactly equal to 1 we're going to use the second piece which is the 2X plus 1 function but if it were to go through X equals 1 what function value would it have? Well you actually get F of 1 which is 2 times 1 plus 1. That means if X equals 1 is an allowable point that you can use to graph the second piece it should go through the point 1, 3 which is right here but because you cannot use X equals 1 for the second piece that means we're actually going to put an open circle at that point to indicate if it were to include that it would be starting right there but we're not going to include it because X value can only be strictly greater than 1 and then we're going to finish that piece using prior knowledge about this equation which we know is a linear equation with a slope of 2 and Y intercept of 1.

For a line with the slope of 2 we're going to go up two units and run over to the right one unit to land at our second point. So two units, one unit, one there and up and then to the right one unit to give us that second point there and I've already connected those two point to get our straight line like that. So the general rule of thumb as you're graphing piecewise functions the first set is to identify your splitting points and then from that point on you want to do function evaluations on that left piece and the right piece, well sometimes could be multiple pieces not just the two pieces but you want to figure out the function values on each of those pieces and label it accordingly depending on whether or not that point is included in your graph you can
finish each piece using prior or known knowledge about the curves. For this next example let's try to think the opposite way.

Here I'm going to give you a graph. I want you to find the formula for the function that looks like to following. The strategy is to first identify points where it changes to a curve. Start from the left side of the real number line. You notice that the first graph we encounter is a horizontal line and it stopped being a horizontal line as soon as we hit the point X equals negative 1 and then it's going to continue on in the pattern of a straight line right here and stop at the point X equals 2 and once you reach that point it's just going to continue on with the horizontal line for the rest of the time. For this function we're going to have three pieces to it. The first piece here which to be determined later we know that it's going to be good until X equals negative 1, so everything to the left of that is good until negative 1 so the domain for that first piece is all X values that is less than or equal to negative one. And just to reiterate you need to include the equal sign here because it is a solid dot at this point. For the second piece it's good between negative 1 and 2 inclusive on the 2 but exclusive on the negative 1.

For the domain for the second piece here is going to be X greater than negative 1 and less than or equal to 2. We're going figure out what the actual function is later. And the domain for this last piece is going to be everything greater than 2. It's strictly greater than because it's an open circle here. Now we're ready to actually figure out what the function description is for each piece. We start with the easiest one here which are the horizontal lines so these are just the constant numbers. Now the first piece is going to be looking at the Y value here for that first horizontal line is going to be 1. This third piece here is going to be easy as well because it's also a horizontal line and then it has a fixed Y value at negative 2. The middle piece is the one that actually needs a little bit of work. You need to figure out a slope and also the Y intercept. The Y intercept is easily available because it is where the line intersects the Y axis which is at 1. You can figure out the slope by traveling from one point to the next. Now going from this point for example to this point here we first go down one unit and then to the right one unit so that corresponds to a negative 1 over 1 slope which is negative 1. All together we have a negative X plus 1.

So to recap the general strategy to figure out a function description for piecewise graphs is to first identify points where splitting happens and to figure out the domain for each piece first and then go backwards to figure out the function description. Next let's look at how we model with piecewise function. We do this through an example here. The cost of long distance daytime phone call from Toronto, Canada to Mumbai, India is 69 cents for the first minute and 58 cents for each additional minute or part of a minute. Draw the graph of the cost of function C in dollars of the phone call as a function of time T in minutes. As a rule of thumb before we start graphing we should figure out the domain for the function that is in question here. If we let C of T be the cost for T minutes and because T is greater than zero the domain for the cost function is the set of all T values such that T is greater than zero. That means we can only use positive T values to construct points for the graph. Well just hypothetically think about what possible values we're looking at here.

Start with say, one minute. How much do you pay? Well if it's within the first minute that means we pay 69 cents and remember you're going to graph in terms of dollars so you're going to
convert the 69 cents to 0.69 dollars. Well what about if you just make a call for 30 seconds? How much do you pay? Well the 30 seconds are still within that first minute of call time which means you also pay 69 cents for that. Now what about zero? If you didn't call at all or if you didn't call then you didn't pay anything but if you just call for one second, how much would you pay? Well you still pay 69 cents for that. What it means that as long as the T value is less than or equal to 1 you will pay 69 cents for that.

If you called for two minutes how much do you pay? That's going to be the 69 cents from the first minute plus the 58 cents for that second minute or that additional one minute which amounts to $1.27. What about if you make a phone call that's a minute and a half? How much do you pay? Well if it's a minute and a half that means you pay 69 cents for the first minute but for that 30 seconds of additional time you would still pay the 58 cents which means you still pay the same thing, $1.27. So what that means is for any T value that is less than or equal to 2 greater than 1 you would pay $1.27. Let's continue this exercise for just one more time. You now call for three minutes. How much do you pay? Well, it's going to be 69 cents for the first minute plus two additional minutes from that and that's 2 times the 0.58 which gives you $1.85. So if you were to call for 2 minutes and one second you would still pay the same thing because we always round up. All right so this number is good for any T value that is less than or equal to 3 but greater than 2. What do you think the next one is the going to be? Pause the video and do this yourself. And I got for T value between 3 and less than or equal to 4, I get $2.43. Now let's practice.

It's always a good idea to lay down your corner system and label your units accordingly. The horizontal axis here represents the T value and we're doing this in minutes. The vertical axis represents the cost which is represented by the variable C and we're doing this in dollars. The first piece right here is good for T value between zero and 1 inclusive on the 1 but can exclusive on the zero. So I'm going to put an open circle on the zero and a closed circle on the 1. This is when T is equal to 1. This is when T is equal to zero and the value is 69 cents which is 0.69 if you consider this in dollars, so any time you're calling between zero minute and the one minute you pay a flat rate of 69 cents and that's what that piece represents. Pause the video and try to finish the rest graph. I get something that looks like this. For the second piece here it's an open circle on the left and it's a closed circle on the right and the value is at 1.27. For the third piece here the Y value is at 1.85 and at the fourth piece the value is at 2.43. The reason we why we don't get a continuous function is because each partial minute gets rounded up to a minute. This is actually called step function with job discontinuity.