STANDARD COURSE OUTLINE

California State University, Long Beach
College of Natural Science and Mathematics
Department of Mathematics and Statistics

I. General information

A. Course number: Math 309

B. Title: Complexity and Emergence

C. Units: 3

D. Prerequisites: GE foundation; upper-division standing

E. Responsible faculty: Scott Crass

F. Prepared by: Scott Crass

G. Date prepared: September 2018

II. Catalog description

Prerequisites: GE foundation, upper-division standing. Introduction to complexity science. Qualitative and computational exploration of emergent properties in dynamical systems, fractals, algorithms, networks, self-organizing behavior, and selected topics.

Students must have scored 11 or higher on the GWAR Placement Examination or successfully completed the necessary portfolio course that is a prerequisite for a GWAR Writing Intensive Capstone. (Lecture, 3 HRS) Grade/Credit-No Credit

III. Curriculum justification

Over the last several decades, the science of complex systems has developed into a mathematically rooted framework for interrogating a broad variety of problems; examples include biological and ecological processes, social interactions, neurological structures, and economic behavior. Complexity and emergence provide a unifying conceptual scheme for studying phenomena that follow simple rules that govern the interactions among elements and yet lead to highly complex results. Indeed, complexity science can be thought of as a means of merging seemingly distinct fields. A flock of birds and a group of traders in a financial market and an ensemble of proteins in a cell can exhibit behavior that gives rise to similar properties. Knowledge of complexity theory deepens a student's level of understanding and especially prepares her to connect topics across distinct disciplines, such as the mathematics of dynamical systems and the behavior of a social or biological network. The mathematics department has no course for either majors or non-majors that treats this new and active field of study. To my knowledge, there is no comparable course offered at the university.

The course also provides a setting in which students explore deep ideas through the medium of writing. By composing essays in response to queries and by developing an independent project, they are challenged to write precisely as well as creatively.

GE category justification, B2

Computational skills. Explorations that take place in and out of class take students through activities such as producing and analyzing graphs, cellular automata, and networks.

Example. Use *NetLogo* to examine a system with a large number of microscopic particles in order to see how data that arise from the phenomena indicate the appearance of emergent macroscopic properties such as energy and entropy.

Methodological approach to problem-solving. Students conduct systematic experiments and simulations that are *agent-based* in order to develop questions and problems that arise in complex systems. Analytical tools are developed that provide insight into how such a problem can be fruitfully examined. This activity is at the core of the query-explore approach to learning.

Example. Simulate the phenomenon of a sandpile that undergoes gradual addition of grains until a column of grains collapses. This experiment exhibits a type of *self-organized* system that undergoes *critical* behavior, a familiar example of which is phase-change such as liquid water freezing.

Application of mathematical concepts. Mathematical ideas and theory underly everything in the course—from dynamics to randomness to algorithms to network structures.

Example. Students construct networks in a variety of ways and then compare what macroscopic properties emerge from the different construction processes.

IV. Measurable student learning outcomes

Math 309 emphasizes careful experimentation and its critical appraisal. Independent and collaborative analytical thinking manifested in writing and speaking are essential for success.

A. Inquiry and analysis

- Students investigate complex systems by means of a query-explore approach. That is, they begin by considering a question that arises in a mathematical or natural setting and then creatively and carefully develop a way of thinking that conveys insight into the problem.
- Through reading and reflecting upon a broad range of material, students adopt a mathematical point of view when looking at a question or problem in nature or society.
- Collaborative work both in and out of class enhances student understanding by promoting dialogue as well as concise, clear, and insightful explanations.
- In a course project, students undertake a significant piece of independent thinking and writing.

- Overall, the course challenges students to use a mathematical perspective creatively in analyzing questions and synthesizing plausible answers. Explorations both in and out of class assess the degree to which students develop such a perspective.

- In this kind of exploration students refine the original question in order to arrive at a new query the exploration of which leads to further questions and exploration. Students thereby engage in a virtuous cycle of inquiry.

- Students are challenged to explore ways in which mathematical understanding can inform their studies in a chosen field as well as broadens their appreciation of mathematics.

- Students work in an environment in which they can see mathematics as an experimental intellectual activity that often arises from questions and problems in a variety of disciplines rather than as a body of knowledge that's applied to such questions and problems.

- Among students with various interests, the course promotes a conversation that realizes mathematical pursuits as experiential and experimental.

B. Quantitative reasoning

- Students conduct experiments of various types that are both physical and computational. They generate data sets that they can represent using graphs and tables from which systemic properties can be extracted. Such work is an important way in which emergent behavior can be seen.
- In lab reports, students describe the results of simulations. These outcomes are typically in a quantitative form in which a relationship among variables can be determined.
- The class explores dynamical systems, particle systems, cellular automata, evolutionary algorithms, and networks. To understand these topics students acquire numeracy at a somewhat advanced level.

C. Written communication

- Students write at length and in depth about their understanding of complex systems and emergent phenomena.

- The course's writing requirement is substantial and promotes clarity, insight, and creativity. By writing essay-type responses to questions that explore issues ranging from the technical to the speculative, students formulate arguments and articulate hypotheses.

- Students' writing, at times, treats philosophical issues and, at others, develops somewhat technical arguments that support a claim.

- Students also describe the process of discovering something through systematic experimentation.

- Finally, students compose a substantial paper that examines some aspect of the vast topic of complexity science. The goal is to foster analytical clarity and insightful perspectives—in thought as well as word.

To allow for reflection on returned, critiqued work and for revision of past and current writing, assignments is spread over the semester. Each of two essays requires 1000-1500 words and the course project calls for a 1500-2000 word paper. Seven lab reports are each 200-300 words. In a preliminary essay submitted during the second week, students respond to an open-ended query that bears upon the first several class discussions. The instructor comments on the preliminary essay by the third week. Students whose preliminary essay indicates a need for special assistance are referred to suitable tutorial resources.

Students must have scored 11 or higher on the GWAR Placement Examination or successfully completed the necessary portfolio course that is a prerequisite for a GWAR Writing Intensive Capstone.

Writing support

- A style and content guide (see below) describes a number of important issues that students should take into consideration as they compose essays and project reports.
- Before each essay is due, class time can be dedicated to writing issues. Working in small groups, students can develop outlines for an essay that are informed by the style and content guide. The instructor actively participates in the group activity by helping to focus the conversation on key features of style and structure. Intermittently, the instructor draws the entire class's attention to important issues that emerge from the group discussions.
- **Style and content guide.** These guidelines are subject to organic development as directed by the varieties of response on the part of students.

  Theme. In *one or two sentences*, you should be able to describe what the point of the essay is. What justifies the existence of any sentence in your essay is that it contributes to the development of the theme.

  Concision. This is a goal of the *final* draft. In early drafts, write effusively—get ideas out as text. After you have a sense of how the writing is developing, you can eliminate inessential words/phrases/sentences in later drafts.

  Flow. In some way (whether obvious or subtle), each subsequent sentence and paragraph should stem from as well as extend beyond the preceding text. An outline can capture the large-scale development.

  Argument. In scientific writing, you're usually trying to develop a case that supports a specific and precisely stated claim. There are several crucial features to such an undertaking.

    Coherence. How well do the text and illustrations fit together? Is it clear that a statement provides support for another? What entitles you to make that statement? What are you assuming? Have you made it clear that you're making those assumptions?

    Hypothetical reasoning. Some of the most important words in scientific writing are 'suppose,' 'imagine,' and 'consider.' Describe the course of a thought experiment: Suppose X occurs. What consequences would or might follow?

Use of examples. Describing or developing a topic that's somewhat or highly abstract in terms of a specific example or case study can be an effective technique. When you formulate an idea in concrete terms you provide a way of understanding the idea not only to the reader but to yourself as well.

In pursuit of these SLO goals, students:

- collect and compare a variety of cases where complex phenomena arise in nature and society;
- analyze and classify their collected examples according to a mathematical understanding of complexity;
- conduct and describe conceptual and computational experiments that involve structures and processes that realize complex behavior;
- examine and connect modes of inquiry and instances of complex structures and behavior across disciplines in natural and social sciences;
- critically articulate and discuss concepts and theories found in an intellectually rigorous collection of reading materials;
- experiment with computational modes of thinking;
- formulate and prepare independent research on historical or contemporary sources.

E. Measurement of outcomes

To measure achievement as well as to promote learning, students engage in a variety of activities.

- Classroom discussion: small-groups and class-wide.
- Essays: Three assignments (1000-1500 words each).
- Computational explorations.
- A substantial course project resulting from independent research (10-15 page paper, class presentation).

V. Outline of subject-matter: Course itinerary

Weeks	Theme	Topics	Coursework
1	Introductory exploration	What is complexity?	Preliminary essay topic distributed
2			Preliminary essay due
2-4	Iteration	Dynamical systems, fractals, tipping points	First essay topics distibuted
5		First essay workshop	First essay due
5-6	Information	Randomness, entropy	
7-8	Evolution	Selection, adaptation, diversity	
9-10	Algorithms	Feedback, genetic algorithm	Second essay topics distributed
10		Second essay workshop	
11-12	Networks	Social structures, polyhedra food webs	Second essay due
			Project prospectus due
13-14	Self-organization	Cellular automata, agent-based models,artificial life	
15	Global reflections	Third essay workshop	
16	Complexity Fair	Projects	Project paper due

VI. Methods of instruction

Class-size is restricted to a maximum of 30. A typical class session is a mix of small group activities, class discussions, and lectures. Students regularly and collaboratively utilize computational tools in order to explore complex processes. They have regular assignments of selections from a rigorous collection of readings. Class discussions and essay questions call on students to analyze critically and creatively what they read and compute. Other essay questions as well as the course project prompt students to think analytically and write with insight.

VII. Extent and nature of technology use

The class frequently uses appropriate software for purposes of running simulations and computational experiments. The main tool is *NetLogo*, a user-friendly yet powerful application that allows students to get started quickly. More advanced students can use the program (or another) to write their own code.

VIII. Information about textbooks/readings

One possibility is for students to explore an intellectually broad and rigorous collection of readings. A suitable choice for a text is Melanie Mitchell's *Complexity: A Guided Tour*.

IX. Instructional policy requirements

Instructors may specify their own policies with regard to plagiarism, withdrawal, absences, etc., as long as the policies are consistent with university policies. It is expected that every course will follow university policies on attendance, syllabi, final grades, grading procedures, and withdrawals.

All sections of the course will have a syllabus that includes the information required by the syllabus policy adopted by the Academic Senate. Instructors will include information on how students may make up work for excused absences. When class participation is a required part of the course, syllabi will include information on how participation is assessed. When improvement in oral communication is an objective of the course, syllabi will include a rubric for how oral communication is to be evaluated.

XI. Bibliography

Complexity—books

C. Adami. *Introduction to Artificial Life*. Springer (1997).

R. Axelrod. *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration*. Princeton (1997).

D. Byrne and G. Callaghan. *Complexity Theory and the Social Sciences: The State of the Art*. Routledge (2013).

L. Fisher. *Perfect Swarm: The Science of Complexity in Everyday Life*. Basic Books (2011).

C. Gros. *Complex and Adaptive Dynamical Systems, a Primer*, 4th edition. Springer (2015).

J. Holland. *Emergence: From Chaos To Order*. Basic Books (1999).

S. Johnson. *Emergence: The Connected Lives of Ants, Brains, Cities, and Software*. Scribner (2002).

D. Meadows. *Thinking in Systems: A Primer*. Chelsea Green (2008).

J. Miller and S. Page. *Complex Adaptive Systems: An Introduction to Computational Models of Social Life*. Princeton (2011).

M. Mitchell. *Complexity: A Guided Tour*. Oxford (2011).

S. Page. *Diversity and Complexity*. Princeton (2010).

D. Sornette. *Critical phenomena in natural sciences: Chaos, fractals, self-organization, and disorder*. Springer (2006).

P. Turchin. *Historical Dynamics: Why States Rise and Fall*. Princeton (2003).

D. Watts. *Small Worlds: The Dynamics of Networks between Order and Randomness*. Princeton (2003).

Complexity—articles

W. B. Arthur. "Complexity economics: A different framework for economic thought" (2013).

G. Hardin. "The tragedy of the commons." *Science* (1968).

M. Newman. "The structure and function of complex networks" (2003).

A. Provata, I. Sokolov, and B. Spagnolo. "Ecological complex systems." *European Physical Journal B* (2008).

T. Schneider, "Information theory primer" (2002).

W. Weaver, "Science and complexity." *American Scientist* (1948).

Writing

S. Pinker. *The Sense of Style: The Thinking Person's Guide to Writing in the 21st Century.* Penguin (2015).

W. Strunk and E. White. *The Elements of Style.* Longman (1999).

W. Zinsser. *On Writing Well.* Harper (2006).

XII. Student-level assessment

The exact set of course assignments can vary depending on the instructor. Appropriate assignments include reading, essays, exams, oral presentations, and a course project.The following assignments provide an example of how grades are determined.

Essay 1	15%
Essay 2	20%
Lab journal	25%
Project	30%
Contribution to class activities	10%

XIII. Course-level assessment

Learning outcomes are assessed with a variety of instruments including contribution to class activities and discussions, essay writing, independent project development and presentation.

XIV. Consistency of SCO standards across sections

The course coordinator will review the SCO and offer advice and materials to any faculty member new to teaching the course. All future syllabi will conform to the SCO. The course coordinator may offer or require regular review of instructors' course materials as well as anonymous samples of student work.

Contact.

Scott Crass, `scrass@csulb.edu`, 54758

Syllabus

Overview

Welcome to *Complexity and Emergence*!

The science of complex systems has developed into a framework rooted in math for interrogating a broad variety of problems; examples include biological and ecological processes, social interactions, neurological structures, and economic behavior. Complexity and emergence provide a unifying conceptual scheme for studying phenomena that follow simple rules that govern the interactions among elements. Knowledge of complexity theory will deepen your level of understanding and especially prepares you to connect topics across distinct disciplines, such as the mathematics of dynamical systems and the behavior of a social or biological network.

Geometry and Symmetry Project

Our course is running as part of the **Long Beach Project in Geometry and Symmetry**, an initiative that promotes thinking that's rooted in perception and experimentation. The project's centerpiece is *The Geometry Studio* which we'll be using as a classroom. Your critical comments—signed or anonymous—on the project or studio experience are welcome at all times.

Getting involved

A typical class session will consist of experiments and explorations that are part of a developing class discussion. You'll work in groups of two or three. Following group work we'll discuss matters as an entire class. The key to success in this course is **initiative**—a willingness to try things and contribute.

Much of our work will be *open-ended* in the sense that you won't figure out something *completely*. Chances are that I won't have all the answers either. Deep thinking really works this way—you're able to make sense of some things out, but there's still more to understand. This is like hiking in the mountains—you reach one peak and gain a nice view, but there are more peaks around you.

Lab explorations

Every two weeks class will meet in a computer lab to explore models that simulate the behavior of complex systems. We'll use the program *NetLogo* which has a user-friendly interface and requires no background in programming.

Reading, writing, and speaking

Text. Melanie Mitchell, *Complexity: A Guided Tour*

Essays. There will be regular assignments of reading from the text. Exploratory exercises and essay questions will be distributed around weeks four and ten. You'll have two to three weeks to write up responses to selected items. The content of this material will depend somewhat upon the directions taken by the course. However, the overall emphasis will be on continuing the explorations we conduct in class. A typical assignment will consist of questions that require essay responses (1000-1500 words) that students write individually. The questions will address issues that arise in the readings and class activities. Written work will be submitted and returned with comments that should inform future essays.

Late essays will be accepted for 90% credit up to **two weeks** after the original deadline.

Lab journal. Each lab exploration will be guided by a handout. You will be asked to conduct experiments and then write brief journal entries (200-400 words) regarding the outcomes. You're encouraged to work in teams (two is best, three is okay). A team should submit one journal entry.

Late entries will be accepted only in case of medical or personal emergency.

Project. A third piece of coursework is the written work and presentation that result from the course project. Emphasis will be placed on clarity of exposition, insightful perspectives, and creativity. A list of sample topics will appear mid-semester, but you're encouraged to originate a project. Please chat with me if you're wondering about a topic or struggling with finding one.

Teamwork (preferably, two members) is encouraged. Submit one paper for the group. On the day of the final exam, we'll hold a **Complexity Fair** in which projects will be displayed and explained.

On writing

Everything that you submit should be written in concise, clear sentences. Experiment with various styles in developing one that works. A style sheet and content rubric is available on the class website. For further help with writing, take a look at

S. Pinker. *The Sense of Style: The Thinking Person's Guide to Writing in the 21st Century.* Penguin (2015).

W. Strunk and E. White. *The Elements of Style.* Longman (1999).

W. Zinsser. *On Writing Well.* Harper (2006).

Prior to the submission of each essay assignment, a class session will be dedicated to a **writing workshop** in which preliminary drafts will be exchanged, read, and critiqued. We'll also discuss issues as a class.

WWW

Materials related to the course (course description, assignments, reference materials) will appear at the class website, a link to which appears at

`geomsymm.cnsm.csulb.edu`

Please make recommendations for things that you'd like to see on the site.

Planned itinerary

- What is complexity?
- Dynamical systems, fractals, chaos
- Information, randomness, entropy
- Evolution, selection, adaptation
- Algorithms
- Self-organization
- Networks

Assessment

Grades will be determined by the following factors.

Essay 1	15%
Essay 2	20%
Lab journal	25%
Project	30%
Contribution to class activities	10%

Here's a *rough* indication of how I'll assign grades. These are **minimum** standards. The actual boundaries between grades might be lower than these, but won't be higher.

85-100%	A
75-85%	B
65-75%	C
50-65%	D

To each individual part of your work I assign a mark 0-10. See below for an *indication* of what these marks mean.

10	Clear, elegant, mathematically and scientifically correct, shows depth of understanding, insight, or creativity
9	Clear, shows understanding and some elegance, insight, or creativity; mathematically and scientifically correct
8	Mathematically and scientifically correct, little elegance, insight, or creativity
7	Mostly mathematically and scientifically correct; little elegance, insight, or creativity
6	Some significant misconceptions
5	Quite significant misconceptions
0-4	Deep misconceptions—shows little effort.

Let me know if you're happy or unhappy about something.

Key to comments on marked papers

a	This needs a supporting **argument**.
a?	What's the **argument**—the line of reasoning—here?
d	**Describe** what's going on here.
e	**Explain** what you're doing here.
f↓	Text does not **flow** well.
h?	**How** did you get this?
i	**Illustrate** what your talking about—give an example, a picture, etc.
p	A **picture** would help here.
s	This is not a **sentence**.
w	**Wording** is awkward, confusing, etc. Meaning is unclear.
y?	**Why** is this so? What's the connection to what you've already said?
!	Very nice. Something especially clear, insightful
?	What this means or what you're doing is **unclear**. Where does this come from?
X	Something's wrong here—in concept or calculation.
$\surd$	This is right—you have the idea.

Sample project topics

Bear in mind. The quality of a project is directly related to its having **substantive content** and a **well-defined**, **narrow focus**. The following are intended to be **suggestive**. Most topics need a narrowing of focus.

- Work out an example of a complex system—real or imagined—where the "whole is greater than the sum of parts" has clear application. Discuss how you can see this phenomenon in clear, quantitative terms. What are the parts? How do they "add up?" What's the whole?
- Discuss the claim that a large city has a fractal-like structure. Identify agents and interactions between them. What processes allow such a structure to emerge? Do fractal properties play a role in a city's functioning?
- Consider a single particle (atom, molecule, say) inside a container. Now add a second particle identical to the first. Describe a property of the 2-particle system that requires the activity of *both* particles. That is, a property that neither particle has individually. What about a property of a three-particle system that's not a property of any of the individual particles nor of any of the pairs of particles. Can this process continue indefinitely? Emergence depends on collective behavior, but how much of a collection of agents is enough?
- Consider or create a piece of art in which you see complex form or behavior. What are the agents in the piece and how do they interact? How does an awareness of emergent properties deepen your understanding of the work?
- Investigate the formation of *emergent structure* produced by bees building a hive (or a comparable phenomenon). What rules do the individual agents follow? How do they interact? What effect does a small change in the rules have on the structure that emerges?
- Gather data on the movements of huddles of emperor penguins. By prescribing rules for each bird to follow, devise an agent-based dynamical model that captures the penguins' motion. It might be a good idea to begin with a small number of agents and then try to increase the number.
- Select, modify, or create a *NetLogo* model and explore its behavior. Devise and conduct experiments that provide insight into the model's emergent properties.
- Set up a computer model that emits a flash (or flashes a dot) at several locations (say, at three points). How does the flash rate and sequence flashes affect what the eye perceives as the flashing occurs? Does the position of the points matter (line segment or triangle, in the case of three points)? What about periodicity or randomness in the sequence of flashes?
- Design a network that synchronizes a behavior (as in firefly flashing). What structures in the network are crucial for the synchronization? Simulating the behavior in a *NetLogo* model would be interesting.
- Explore "chromatic cellular automata" whose states are a color from a fixed set (three colors, say) and a neighborhood rule for determining the next state. It would be nice to see it simulated in *NetLogo* (or other system).

- Develop a *NetLogo* (or other system) model of a cellular automaton that generates concentric circles of states.
- Work out a *NetLogo* model of an evolutionary biological process in which interactions among agents conform to a game-theoretic payoff rule. You could give the interaction dynamics spatial properties—a model of an eco-space. How and where the interactions occur could be an adjustable parameter. Also, a method of mutation introduction is needed; parameters could specify when, where, and to what degree mutations occur.

  Simulate interactions over time and multiple generations. How do the populations of the respective phenotypes behave over time? How does the outcome depend on the initial conditions—number of mutants, mutation rate, etc.
- Investigate the dynamical behavior of a grid of springs that are connected to each other as well as to a boundary shape. (The investigation can be carried out by constructing a physical system, a virtual system—simulation, or by theoretical development.) Examples of grid arrangements are triangles, squares, or hexagons. Identify emergent properties or behavior that results from disturbing the grid away from equilibrium. Are there natural systems that behave in similar ways? What role does the density of springs play in the emergence of dynamical properties? How are emergent properties affected by making changes—small ones being the most interesting—in the boundary shape?