STANDARD COURSE OUTLINE

California State University, Long Beach
College of Natural Science and Mathematics
Department of Mathematics and Statistics

I. General information

A. Course number: Math 303

B. Title: Reflections in Space and Time

C. Units: 3

D. Prerequisites: GE foundation; upper-division standing

E. Responsible faculty: Scott Crass

F. Prepared by: Scott Crass

G. Date prepared: September 2018

II. Catalog description

Prerequisites: GE foundation, at least one GE Explorations course, upper-division standing. Subtitle: Explorations in symmetry and imagination. An experimentally-driven investigation of the mathematical nature of symmetry and patterns. Considers the pervasive appearance and deep significance of symmetry and patterns in art and science.

Students must have scored 11 or higher on the GWAR Placement Examination or successfully completed the necessary portfolio course that is a prerequisite for a GWAR Writing Intensive Capstone. (Lecture, 3 HRS) Grade/Credit-No Credit

III. Curriculum justification

As the mathematics department's first GE capstone course, Math 303 has attracted students of art, biology, chemistry, communication, computer science, economics, engineering, English, film, finance, geology, journalism, linguistics, mathematics, physics, political science, and psychology. Outcomes indicate that students have experienced a high level of intellectual challenge and enrichment through their exploration of the deep concepts of symmetry and pattern. A mathematical perspective deepens their understanding of symmetry and patterns as they appear in other fields. Interestingly, many students come to regard mathematics in a new and more favorable light.

The course also provides a setting in which students explore a deep idea through the medium of writing. By composing essays in response to queries and by developing an independent project, they are challenged to write precisely as well as creatively.

GE category justification, B2

Computational skills. The class develops and utilizes an algebraic mode of description in order to formulate a mathematical characterization of symmetry. This account forms the foundation of the course and is applied throughout in order to develop models of processes in physics, chemistry, biology, and art.

Example. In a classroom group-based activity students determine the symmetries of various objects–such as an equilateral triangle and tetrahedron. This investigation leads to a discussion of how to describe, in algebraic terms, transformations such as reflections, rotations, and translations. Relationships among these transformations is also studied in detail.

Methodological approach to problem-solving. The course challenges students to use a mathematical perspective on symmetry to creatively analyze problems and synthesize answers. They see and experience how the idea of symmetry can be employed to devise and solve scientific and artistic problems.

Example. Using balloons to model an attraction-repulsion model of molecular interactions, groups of students explore what stable structures appear when clusters of particles form.

Application of mathematical concepts. The class experiments with perceptual modes of thinking. Students determine and explain the significance of symmetry in scientific and artistic work.

Example. Collision experiments provide a natural setting for asking about system properties that don't change when an interaction (in this case, a collision) takes place. These cases illustrate a very general and important framework for processes that involve input-interaction-output behavior.

IV. Measurable student learning outcomes

Math 303 involves creative imagination and its critical appraisal. Independent and collaborative analytical thinking manifested in writing and speaking is essential for success. The general course goals are:

- To challenge students to explore ways in which mathematical understanding can inform their studies in a chosen field as well as promote their appreciation of mathematics.
- To provide students with an environment in which they can see mathematics as an experimental intellectual activity that often arises from questions and problems in a variety of disciplines rather than as a body of knowledge that's applied to such questions and problems.
- To promote, among students with various interests, a conversation that realizes mathematical activity as experiential and experimental.

Math 303 students make observations, conduct experiments, and participate in developing a theoretical understanding of symmetry toward which the results point. From a mathematical vantage point they explore some deep issues in art, biology, chemistry, and physics. Their approach to problems draws on methods and questions specific to the subject. Indeed, determining what constitutes a problem is one of the most fundamental issues within a discipline. What role do laboratory and thought experiments play in determining a physical law? How do the hypotheses of atomic and molecular structure inform chemical analysis and vice versa? To what extent can the structure of organisms be understood theoretically? What character does a work of art acquire by employing symmetrical or asymmetrical structures? How do you go about conducting and analyzing an experiment in mathematics? Furthermore, philosophical inquiry runs through the course. For example, there's a question of *realism*: Is symmetry *in* an object or is it a manifestation of our descriptive apparatus? Of course, each of these vast subjects can be pursued at much greater length than this course can accommodate. However, symmetry and pattern provide a conceptual lens that focuses thought so that it can cut deeply as well as broadly.

A. Inquiry and analysis: A mathematical point of view

- Examine and connect modes of inquiry and instances of symmetric structures across disciplines.
- Critically articulate and discuss concepts and theories found in an intellectually rigorous collection of reading materials.
- Through reading a broad range of material, students learn to adopt a mathematical point of view when looking at a question or problem in science or art.
- Collect and compare a variety of cases where symmetry and patterns arise in science, art, and general experience.
- Analyze and classify their collected examples according to a mathematical understanding of symmetry and pattern.
- Collaborative in-class activities and presentations enhance student understanding by promoting dialogue as well as concise, clear, and insightful explanations.
- In a course project, students undertake a significant piece of independent thinking and writing.
- Formulate and prepare independent research on historical or contemporary sources.

B. Quantitative reasoning

- Conduct and describe thought experiments that involve structures and processes that realize symmetric properties.
- Interpret and apply a mathematical description of symmetry in a variety of conceptual settings.
- Measure the degree of symmetry a process or object possesses.

C. Written communication

- Students write at length and in depth about the concept and significance of symmetry and pattern.
- At times, students' writing treats philosophical issues and, at others, develops somewhat technical arguments that support a claim.
- By writing essay type responses to a variety of questions—from open-ended philosophical issues to problems of a more technical nature, students formulate arguments and articulate hypotheses.
- Students describe the process of discovering something through systematic experimentation.
- Students compose a substantive paper that independently examines some aspect of the vast topic of symmetry and pattern. The goal is to foster analytical clarity and insightful perspectives—in thought as well as word.

To allow for reflection on returned work and revision of current writing, assignments are spread over the semester. As for length, each of the three essays require 1000-1500 words and the course project calls for a 2000-3000 word paper. In a preliminary essay submitted during the second week, students respond to an open-ended query that bears upon the first several class discussions. The instructor comments on the preliminary essay by the third week. Students whose preliminary essay indicates a need for special assistance are referred to suitable tutorial resources.

Writing support

- A style and content guide (see below) describes some important issues that students should take into consideration as they compose essays and project reports.
- Before each essay is due, a class session can be dedicated to an "essay workshop" in which students exchange preliminary drafts of their work. Working in small groups, readers provide peer-review of a piece of writing by applying the principles articulated in the style sheet and content rubric. The instructor actively participates in the group activity by contributing critical assessment of written material as well as helping to focus the conversation on key features of style and structure. Intermittently, the instructor draws the entire class's attention to important issues that emerge from the group discussions.
- **Style and content guide.** These guidelines are subject to organic development as directed by the varieties of response on the part of students.

  Theme. In *one or two sentences*, you should be able to describe what the point of the essay is. What justifies the existence of any sentence in your essay is that it contributes to the development of the theme.

  Concision. This is a goal of the *final* draft. In early drafts, write effusively—get ideas out as text. After you have a sense of how the writing is developing, you can eliminate inessential words/phrases/sentences in later drafts.

Flow. In some way (whether obvious or subtle), each subsequent sentence and paragraph should stem from as well as extend beyond the preceding text. An outline can capture the large-scale development.

Argument. In scientific writing, you're usually trying to develop a case that supports a specific and precisely stated claim. There are several crucial features to such an undertaking.

Coherence. How well do the text and illustrations fit together? Is it clear that a statement provides support for another? What entitles you to make that statement? What are you assuming? Have you made it clear that you're making those assumptions?

Hypothetical reasoning. Some of the most important words in scientific writing are 'suppose,' 'imagine,' and 'consider.' Describe the course of a thought experiment: Suppose X occurs. What consequences would or might follow?

Use of examples. Describing or developing a topic that's somewhat or highly abstract in terms of a specific example or case study can be an effective technique. When you formulate an idea in concrete terms you provide a way of understanding the idea not only to the reader but to yourself as well.

- **References.**

S. Pinker. *The Sense of Style: The Thinking Person's Guide to Writing in the 21st Century.* Penguin (2015).

W. Strunk and E. White. *The Elements of Style.* Longman (1999).

W. Zinsser. *On Writing Well.* Harper (2006).

D. Interdisciplinary learning

The concepts of symmetry and pattern forge connections between art, mathematics, and science. Artistic and scientific developments produce structures—whether material or abstract—with respect to which considerations of symmetry and pattern arise naturally. With mathematics, we can articulate a theoretical understanding of these fundamental and pervasive ideas.

Methodological practices of observation and experimentation create another unifying thread between art, science, and mathematics. However, the principles that guide empirical investigations vary significantly across disciplines. Stemming from experimental results—both physical and conceptual, basic scientific laws and principles give rise to theoretical frameworks that provide descriptions of and explanations for further experimental phenomena that are both qualitative and quantitative. In physics these interpretive frameworks manifest themselves in ways that are quite different from those arising in chemistry and biology. While there are no laws of art that structure their observations, artists do engage in experimental activity and adjust their work in light of the outcomes. In mathematics, the results of systematic experiments can suggest the "existence" of certain structures as well as the truth of certain claims (theorems).

E. Measurement of outcomes

To measure achievement, students engage in a variety of activities.

- Classroom activities and discussion: small-groups and class-wide.
- Essays: Three assignments, two essays each (2-3 pages each essay).
- A substantial course project resulting from independent thought (5-10 page paper, class presentation).

V. Outline of subject-matter: Course itinerary

Weeks	Topics	Coursework
1	Introductory exploration: What is symmetry?	Collect and discuss examples Preliminary essay topic distributed
2		Preliminary essay due
2-5	Developing a mathematical treatment	First essay topics distributed
5	First essay workshop	First essay due
5-9	Symmetry of physical laws and theories	Second essay topics distributed
10	Second essay workshop	Second essay due
9-11	Molecular symmetry and chemical properties	
11-13	Biological significance of symmetry	Third essay topics distributed Project prospectus due
13-15	Use of symmetry/asymmetry in art	
15	What we've overlooked Third essay workshop	Third essay due
16		Symmetry Fair Project paper due

VI. Methods of instruction

Class-size is restricted to a maximum of 30. A typical class session is a mix of small group activities, class discussions, and lectures. At semester's end, students present their course projects to class. Students have regular assignments of selections from a rigorous collection of readings. Class discussions and essay questions call on students to critically analyze what they read. Other essay questions as well as the course project prompt students to think analytically and write with insight.

VII. Extent and nature of technology use

Occasionally, a data projector and video recordings will be used to illustrate certain patterns and structures as well as to stimulate classroom activities and discussion. The class website

provides access to written and graphical material that can lead students to exploration beyond class content.

VIII. Information about textbooks/readings

A collection of readings covering a wide range of thinking is assigned and available online.

IX. Instructional policy requirements

Instructors may specify their own policies with regard to plagiarism, withdrawal, absences, etc., as long as the policies are consistent with university policies. It is expected that every course will follow university policies on attendance, syllabi, final grades, grading procedures, and withdrawals.

All sections of the course will have a syllabus that includes the information required by the syllabus policy adopted by the Academic Senate. Instructors will include information on how students may make up work for excused absences. When class participation is a required part of the course, syllabi will include information on how participation is assessed. When improvement in oral communication is an objective of the course, syllabi will include a rubric for how oral communication is to be evaluated.

XI. Bibliography

Books

Art

E. Crook, ed. *Fearful Symmetry: Doubles and Doubling in Literature and Film* (1980).
M. Emmer, ed. *The Visual Mind* (1993).
M. Emmer, ed. *The Visual Mind II* (2005).
M. Ghyka. *The Geometry of Art and Life* (1946).
J. Kappraff. *Connections: The Geometric Bridge Between Art and Science* (2001).
G. Keper. *Module, Proportion, Symmetry, Rhythm* (1971).
C. MacGillavry. *Fantasy and Symmetry* (1976).

Biology

R. Jean and D. Barabé. *Symmetry in Plants* (1998).
E. Charnov. *Life History Invariants: Some Explorations in Evolutionary Ecology* (1993).
T. Cook. *The Curves of Life* (1914).
D. Thompson. *On Growth and Form* (1917).

Chemistry

J. Baggot. *Perfect Symmetry: The Accidental discovery of a New Form of Carbon* (1994).
I. Hargittai and M. Hargittai. *Symmetry through the Eyes of a Chemist* (1995).

Multidisciplinary

P Ball. *The Self-Made Tapestry: Pattern Formation in Nature* (1999).
F. Close. *Lucifer's Legacy: The Meaning of Asymmetry* (2001).
D. Crowe and D. Washburn. *Symmetry Comes of Age: The Role of Pattern in Culture* (2004).
G. Fleck and M. Senechal, eds. *Patterns of Symmetry* (1977).
M. Gardner. *The New Ambidextrous Universe : Symmetry and Asymmetry from Mirror Reflections to Superstrings* (1990).
M. Golubitsky and I. Stewart. *Fearful Symmetry: Is God a Geometer?* (1992).
W. Hahn. *Symmetry as a Developmental Principle in Nature and Art* (1998).
K. Mainzer. *Symmetries of Nature: A Handbook for Philosophy of Nature and Science* (1996).
K. Mainzer. *Symmetry And Complexity: The Spirit And Beauty Of Nonlinear Science* (2005)
J. Rosen. *Symmetry Discovered* (1975).
J. Rosen. *Symmetry in Science* (1994).
I. Stewart. *What Shape is a Snowflake?* (2001).
I. Stewart. *Why Beauty Is Truth: The Story of Symmetry* (2007)
H. Weyl. *Symmetry* (1952).

Math

M. Armstrong. *Groups and Symmetry* (1988).
F. Budden. *The Fascination of Groups* (1972).
M. Field. *Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature* (1992).
I. Hargittai, ed. *Fivefold Symmetry* (1992).
L. Kinsey and T. Moore. *Symmetry, Shape, and Space* (2002).

Philosophy

B. van Fraassen. *The Scientific Image* (1980).
B. van Fraassen. *Laws and Symmetry* (1989).
M. Leyton. *Symmetry, Causality, and Mind* (1992).

Physics

B. Bunch. *Reality's Mirror: Exploring the Mathematics of Symmetry* (1989).
R. Feynman. *Six Not So Easy Pieces* (1997).
C. Hill and L. Lederman. *Symmetry and the Beautiful Universe* (2004).
V. Icke. *The Force of Symmetry* (1995).
T. Lee. *Symmetries, Asymmetries, and the World of Particles* (1988).

Periodicals

Leonardo. (Print journal, International Society for the Arts, Sciences and Technology).
Symmetry: Art and Science. (Print journal, International Society for the Interdisciplinary Study of Symmetry).
Visual Math (`www.mi.sanu.ac.yu/vismath/vismath`). (Online journal, International Society for the Interdisciplinary Study of Symmetry).

Video

Arabesques and Geometry (1995).
Understanding Beauty (1995).

XII. Student-level assessment

The exact set of course assignments will vary depending on the instructor. Appropriate assignments include reading, essays, exams, oral presentations, and a course project. The following assignments provide an example of how grades are determined.

Three essays	50%
Course project	40%
Contribution to class activities	10%

XIII. Course-level assessment

Learning outcomes will be assessed with a variety of instruments including contribution to class activities and discussions, essay writing, independent project development and presentation.

XIV. Consistency of SCO standards across sections

The course coordinator will review the SCO and offer advice and materials to any faculty member new to teaching the course. All future syllabi will conform to the SCO. The course coordinator may offer or require regular review of instructors' course materials as well as anonymous samples of student work.

Contact.

Scott Crass, `scrass@csulb.edu`, 54758

Syllabus

Overview

Symmetry and patterns are all around us. Or are they? Do we *discover* that something *has* symmetry or *is* a pattern? Or, is this a matter of our imposing a concept on the world? It must be that both processes are involved. Does an abstract rectangle have the "same" symmetry as a picture frame? Why do we want to see objects *as* symmetric even though they aren't quite? What features do we overlook? What do we perceive in this act of *symmetric-seeing*? What about other senses? Can we hear or feel symmetry?

These questions quickly lead to the heart of philosophical theories of art, perception, mathematics, and ... what else? This course is designed to promote an inquiry-driven conversation among students whose primary interests vary widely. Bringing their own expertise and insight to an exploration of the rich ideas of symmetry and pattern, students will collect, present, and investigate examples having to do with this pervasive aspect of the world. Whether it stems from visual art, music, literature, or a science the challenge for each will be to make the examples accessible to the whole of the class. Such a collaboration and discussion will produce new means of understanding and appreciating symmetry and patterns.

The driving principle of this course is that mathematics both stems from and helps us make sense of our experience. Perceptual and conceptual experimentation will provide its experiential content.

Geometry and Symmetry Project

Our course is running as part of the **Long Beach Project in Geometry and Symmetry**, an initiative that promotes thinking that's rooted in perception and experimentation. The project's centerpiece is *The Geometry Studio* which we'll be using as a classroom. Your critical comments—signed or anonymous—on the project or studio experience are welcome at all times.

Getting involved

A typical class session will consist of experiments and explorations that are part of a developing class discussion. You'll work in groups of 2-4. Following group work we'll discuss matters as an entire class. The key to success in this course is **initiative**—a willingness to try things and contribute.

Much of our work will be *open-ended* in the sense that you won't figure out something *completely*. Chances are that I won't have all the answers either. Deep thinking really works this way—you're able to make sense of some things out, but there's still more to understand. This is like hiking in the mountains—you reach one peak and gain a nice view, but there are more peaks around you.

Reading, writing, and speaking

A preliminary essay will be assigned and collected in the first two weeks.

There will be weekly assignments of readings and three exploratory essays. The overall emphasis will be on organizing and making sense of our experience—observations, experimental activities. A typical assignment will consist of several questions that require essay responses. For each assignment, you will select two items to explore and write about (500-600 words per essay). The questions will address issues that arise in the readings and class-discussions. Written work will be submitted and returned with comments that should inform future essays. The resulting marks will form a significant part of the course grade. (**Late essays** will be accepted for 90% credit up to **two weeks** after the original deadline.)

I urge you to **work with others** and to consider the questions at some length before asking about them in class. I also encourage you to ask about them. You may submit a write-up for the exercises as a group of two. The work represented in what you submit should be **your own.** Everything that you submit should be written in concise, clear sentences. Experiment with various styles in developing one that works.

A third piece of the course-grade will be the evaluation of the written work and presentation that result from the course project. Emphasis will be placed on clarity of exposition, insightful perspectives, and creativity.

Teamwork (preferably, two members) is encouraged. Submit one paper (1500-2000 words) for the group. On the day of the final exam, we'll hold a **Symmetry Fair** in which projects will be displayed and explained.

Course Project

Working individually or collectively students will independently explore some part of the vast world of symmetry and pattern. (Collaborative work in groups of two or three is strongly encouraged.) This is an opportunity for experimentation, hypothesis, construction, and, above all, creativity. The nature of the project can be artistic, theoretical, philosophical, historical, technological, etc. A list of sample topics will appear by the fifth week. A key to a rewarding project is to explore in depth a *narrowly focussed* topic. A broad survey or book report is not suitable. I encourage you to chat with me early and often as you develop your project.

By week eight, you should be considering a project topic. Feel free to discuss your ideas with me.

By week twelve, you should have a fairly clear vision of what you plan to undertake. What will be the focus of your investigation? How will you approach the question, problem, or issue? What means of research—physical or thought experiment, abstract reasoning, sources—will you use?

Each project should be accompanied be a paper that develops and explains the ideas you've explored. If you work in some medium (drawing, painting, sculpture, film, etc.), you should submit an interpretive paper. At the end of the semester we'll have a *Symmetry Fair* when you will display and and discuss your work.

On writing

Everything that you submit should be written in concise, clear sentences. Experiment with various styles in developing one that works. Guidelines to style and content are available on the class website. For further help with writing, take a look at

> S. Pinker. *The Sense of Style: The Thinking Person's Guide to Writing in the 21st Century*. Penguin (2015).
>
> W. Strunk and E. White. *The Elements of Style*. Longman (1999).
>
> W. Zinsser. *On Writing Well*. Harper (2006).

Planned itinerary

Weeks	Topics	Coursework
1-5	What is symmetry? "I can't define it, but I know it when I see it."	Preliminary essay distributed and due 1st essay distributed
5-15	Symmetry in science and art "I can define it, but I don't know it when I see it."	
5-9	What's a physical law? Are physical laws symmetrical? What happens if time runs backwards? Is the universe right or left-handed?	1st essay due 2nd assignment distributed
9-12	Atoms, molecules, and matter Molecular symmetry and chemical properties	2nd essay due
12-14	Growth, form, and life Biological significance of symmetry Phyllotaxis and the golden ratio	3rd assignment distributed
14-15	Use of symmetry/asymmetry in art	3rd essay due
16		Symmetry Fair

WWW

Materials related to the course (course description, assignments, reference materials) will appear on project's website,

`geomsymm.cnsm.csulb.edu`

Please make recommendations for things that you'd like to see on the site.

Assessment.

Grades will be determined by the following factors.

Essays	50%
Course project	40%
Contribution to class activities	10%

Here's a *rough* indication of how I'll assign grades. These are **minimum** standards. The actual boundaries between grades might be lower than these, but won't be higher.

85-100%	A
75-85%	B
65-75%	C
50-65%	D

To each individual part of your work I assign a mark 0-10. See below for an *indication* of what these marks mean.

10	Clear, elegant, mathematically and scientifically correct, shows depth of understanding, insight, or creativity
9.....	Clear, shows understanding and some elegance, insight, or creativity; mathematically and scientifically correct
8.....	Mathematically and scientifically correct, little elegance, insight, or creativity
7.....	Mostly mathematically and scientifically correct; little elegance, insight, or creativity
6.....	Some significant misconceptions
5.....	Quite significant misconceptions
0-4....	Deep misconceptions—shows little effort.

Let me know if you're happy or unhappy about something.

Key to comments on marked papers

a	This needs a supporting **argument**.
a?	What's the **argument**—the line of reasoning—here?
d	**Describe** what's going on here.
e	**Explain** what you're doing here.
f↓	Text does not **flow** well.
h?	**How** did you get this?
i	**Illustrate** what your talking about—give an example, a picture, etc.
p	A **picture** would help here.
s	This is not a **sentence**.
w	**Wording** is awkward, confusing, etc. Meaning is unclear.
y?	**Why** is this so? What's the connection to what you've already said?
!	Very nice. Something especially clear, insightful
?	What this means or what you're doing is **unclear**. Where does this come from?
X	Something's wrong here—in concept or calculation.
√	This is right—you have the idea.