CSULB MATH DAY AT THE BEACH 2019

INDIVIDUAL ROUND, PART 1: MULTIPLE CHOICE – *Write your name and school and mark your answers on the answer sheet. You have 30 minutes to work on these problems. No calculator is allowed.*

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1. How many real numbers x are such that $\sqrt{2 - x/2} = 2 + x$?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

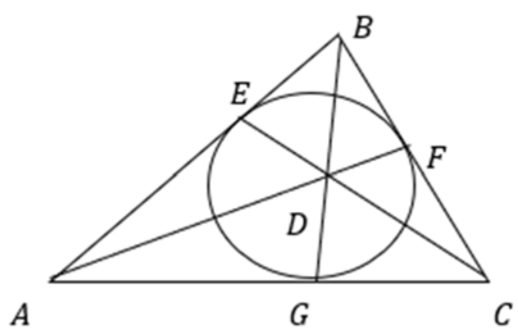
2. A box contains four different pairs of socks. Socks are drawn randomly, two at a time, without replacement, until all eight socks are drawn. What is the chance that all four pairs match?

   (A) 1/8 (B) 1/20 (C) 1/105 (D) 1/24 (E) 1/144

3. What values of x satisfy the inequality $\log_2(x+1) > \log_2 3 + \log_4 25 + \log_8 343$?

   (A) $x \in (104, \infty)$ (B) $x \in \left(-\infty, \frac{7}{22}\right]$ (C) $\left(\frac{4}{3}, \frac{6}{5}\right)$ (D) $(21, \infty)$ (E) $[0, 1]$

4. A circle with center D is inscribed in $\triangle ABC$, $\angle EAD = 15°$ and $\angle GCF = 76°$. Find $\angle DBF$.



   (A) 32° (B) 40° (C) 37° (D) 74° (E) 18°

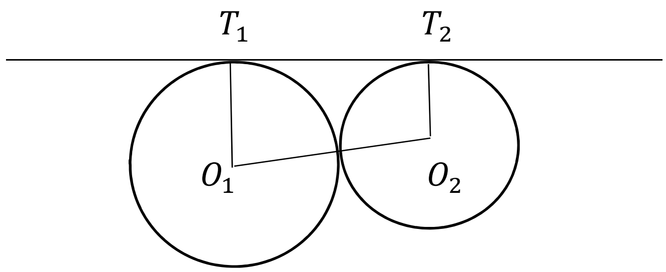
5. Positive integers a and b satisfy $\frac{a}{b} = 0.2019\ldots$. What is the smallest possible value of b?

   (A) 7 (B) 20 (C) 2018 (D) 104 (E) 1104

6. How many solutions of the equation $\sin^4 x + \cos^4 x = 1$ are there in the interval $0 \le x \le 2\pi$?

   (A) 0 (B) 1 (C) 2 (D) 4 (E) 5

7. Two circles of radii R_1 and R_2 are touching externally and have the centers at the points O_1 and O_2. A common tangent line touches the circles at points T_1 and T_2, respectively. Find the area of the polygon $O_1T_1T_2O_2$.



(A) $\frac{1}{2}R_1R_2$ (B) $(R_1+R_2)^2$ (C) $\frac{1}{2}(R_1+R_2)\sqrt{R_1R_2}$ (D) $(R_1+R_2)\sqrt{R_1R_2}$ (E) R_1+2R_2

8. Choose three numbers from the set $\{1,2,3,\ldots,20\}$ so that no two numbers are consecutive. How many different choices are there?

(A) 816 (B) 1298 (C) 905 (D) 2019 (E) 1116

9. Letters $A, B, C,$ and D denote distinct digits between 0 and 9. The sum of four-digit numbers $BACA$ and $CCCC$ equals $DDBAC$. What is the value of $A+B+C+D$?

(A) 8 (B) 12 (C) 15 (D) 18 (E) 13

10. Let $\{x_n\}$ be a sequence such that $x_1 = 2,$ and $(n+1)x_{n+1} = x_n + n$ for all $n \geq 1$. What is x_{2019} ?

(A) $\frac{2018!}{2018!+2}$ (B) $\frac{2019!}{2019!+1}$ (C) $\frac{2019!+1}{2019!}$ (D) $\frac{2018!+1}{2018!}$ (E) $\frac{2018!}{2018!+1}$

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INDIVIDUAL ROUND, PART 2: FREE RESPONSE – *Write your name and school and mark your answers on the answer sheet. You have 20 minutes to work on these problems. No calculator is allowed.*

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11. Let C be a cube with side length of 1. We put C in the Euclidean 3-dimensional space so that one of its main diagonals is contained in the z-axis. Let A be the projection of C to the xy-plane (that is, A is the set of points (x, y) so that there exists z with $(x, y, z) \in C$). What is the area of A?

12. Let P be the point obtained by rotating the point (3, 4) around the point (1,-1) counterclockwise by 90°. Find the coordinates of P.

13. How many integers n with $2 \leq n \leq 750$ can be written as $n = a^b$, so that a and b are integers, and $b > 1$?

14. What is the closest integer to $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2019}$?

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INDIVIDUAL ROUND, PART 3: FREE RESPONSE – *Write your name and school and mark your answers on the answer sheet. You have 20 minutes to work on these problems. No calculator is allowed.*

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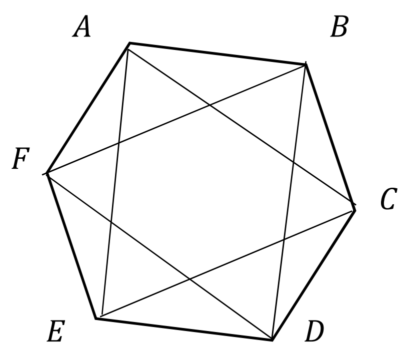
15. Factorial of what number should be removed from the product $(1!)(2!)(3!)\ldots(20!)$ to make the remaining product a square of a positive integer?

16. In a regular hexagon $ABCDEF$ of area 1, the vertices are connected as shown on the picture. Find the area of the smaller hexagon formed in the middle.



17. Find the maximum of the function $y = x^2 - 6|x| + 6$ in the interval $-5 \leq x \leq 5$.

18. Popcorn is packaged in boxes before a baseball game, and in each box the vendor inserts a card of one of the five recent most valuable players (MVPs), picked randomly. You buy four boxes of popcorn. What is the probability that you get the cards for exactly three different MVPs?

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TEAM ROUND – *Write your school name and mark your answers on the answer sheet. You have 30 minutes to work on these problems. No calculator is allowed.*

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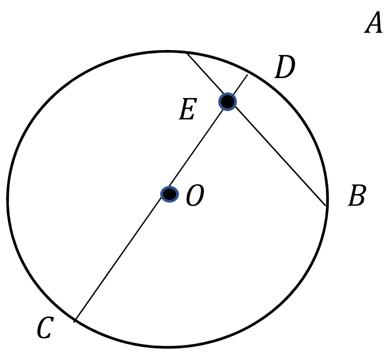
1. How many ordered pairs (a, b) are such that the equation $x^4 - (a + b + 1)x^3 + (a + b + ab)x^2 - abx = 0$ has exactly two distinct solutions?

2. If $0 \le x \le \frac{\pi}{2}$, in what interval does the inequality $\cos^3 x < 3\sin^2 x \cdot \cos x$ hold?

3. In a circle with center O, the cord $\overline{AB}$ intersects the diameter $\overline{CD}$ at the point E. The angle $\angle AOB$ equals $60°$, and the ratio of the lengths $|\overline{CE}|$ and $|\overline{ED}|$ is 19:1. Assuming $|\overline{AE}| < |\overline{EB}| = 19$, find $|\overline{AE}|$. Leave the answer in the form $a + b\sqrt{c}$ for some real $a, b,$ and c.



4. The numbers 2019^k are divided by 7 for $k = 1,2, \dots, 2019$. What is the sum of all residuals?

5. Consider a unit sphere and a point O that lies outside the sphere. Three tangent lines from the point O touch the sphere at the points $A, B,$ and C. The plane ABC intersects the sphere at a circle of radius ½. What is the maximum possible volume of the tetrahedron $OABC$?

6. Let $\mathbb{R}$ denote the set of real numbers. For what values of $x \in \mathbb{R}$ does the inequality $4\cos^4 x + \sin^2 x \ge 3\cos^2 x$ hold?

7. All the three-digit numbers are written in a row: $100101102 \dots 998999$. How many times a two is followed by a zero in this sequence of digits?

8. Represent $\frac{2}{7}$ as the sum of two fractions $\frac{1}{a}$ and $\frac{1}{b}$ where a and b are natural numbers with $a < b$.

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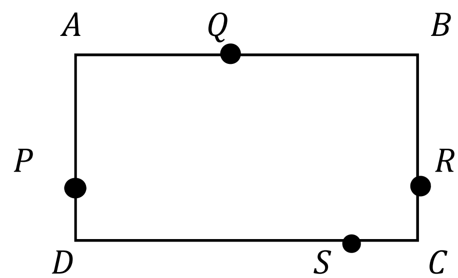
RELAY ROUNDS

Relay 0, Question 1. Compute $4 \times 4 + 4$.

Relay 0, Question 2. Let T=TNYWR. Compute $3T$.

Relay 0, Question 3. Let T=TNYWR. Compute $T/6$.

Relay 1, Question 1. The points P, Q, R, and S are placed on the sides of rectangle $ABCD$ in such a way that $AP = PD$ and $BR = RC$. The area of the polygon $PQRS$ equals 9. Find the area of $ABCD$.



Relay 1, Question 2. Let T=TNYWR. Suppose m and n are prime numbers such that $m + n = T - 6$. Find $m^2 + n^2$.

Relay 1, Question 3. Let T=TNYWR. The area of a rhombus is $T + 6$. If one diagonal is 20 units long, what is the length of the other diagonal?

Relay 2, Question 1. How many positive integer divisors does 1768 have?

Relay 2, Question 2. Let T=TNYWR. How many solutions does the equation $\sin(Tx) + \cos(Tx) = 0$ have in the interval $0 < x < \frac{\pi}{4}$?

Relay 2, Question 3. Let T=TNYWR. What is the sum of the negative integers that satisfy the inequality $2x - 3 > -T - 7$?

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FACEOFF ROUND

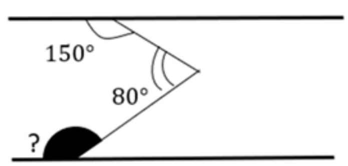
Question 1. What is the largest prime factor of $3 \times 5 \times 7 \times 11 + 2$?

Question 2. Consider a regular n-sided polygon. If the product of the number of diagonals and the measure (in degrees) of its angle equals 1800, what is the value of n?

Question 3. Compute base b if $\log_b 405 = 3 + \log_b 3 + \log_b 5$?

Question 4. Three students are taking a test. Each of them will pass the test with probability 0.7, independently of the others. What is the probability that at least one of them will pass?

Question 5. The two lines are parallel. Given the two angles, find the third one.



Question 6. Find $m^3 + \frac{1}{m^3}$ if $m + \frac{1}{m} = 3$.

Question 7. What is the smallest integer greater than 2 such that the division by each of $3, 4,$ and 5 yields a remainder of 2?

Question 8. Randomly choose a three-digit number (that is, n such that $100 \leq n \leq 999$). What is the probability that the digits of this number are in strictly increasing order? Express your answer as a fraction in lowest terms.

Question 9. Instead of walking in a straight line, a person walked in a perfect semicircle. What percent more of the distance did he walk? Round your answer to two decimals.

Question 10. Define a function f on the set of positive integers by

$$f(n) = \begin{cases} n - 1 & if\ n\ is\ event, \\ n^2 - 1 & if\ n\ is\ odd. \end{cases}$$

Find and list all n such that $f(f(n)) = 8$.

Question 11. There are 2019 balls. 2018 of them weigh 1 gram each, and one of them weighs 2 grams. You have an electric scale that will tell you the total weight of whatever you put on the scale. How many weightings must you perform, at minimum, to be certain to find the heavy ball?

Question 12. Assume $\cos^4\theta + \sin^4\theta = \frac{4}{5}$. Compute $\cos^6\theta + \sin^6\theta$.