1 Individual Round I

1. A bag has some white balls and some red balls. After 7 red balls are removed, the number of white balls becomes 3 times the number of red balls. After 22 white balls are *then* removed, and after that, the number of red balls is 4 times as the number of white balls. How many balls (red plus white) did we start with?
(A) 15 (B) 24 (C) 29 (D) 39 (E) Cannot be determined from the information given

Solution: Let the number of red balls be R and the number of white balls be W and we obtain two equations:

$$\begin{cases} W &= 3(R-7) \\ R-7 &= 4(W-22) \end{cases}$$

Solving the equations we find that

$$\begin{cases} R = 15 \\ W = 24 \end{cases}$$

and we get $R + W = 39$, thus we obtain the answer **(D)**.

2. For all x which the expressions is defined, $\dfrac{1}{\sec x + \csc x} =$
(A) $\dfrac{1}{\cos x + \sin x}$ (B) $\cos x + \sin x$ (C) $\dfrac{\sqrt{2}\sin(2x)}{4\sin\left(x + \frac{\pi}{4}\right)}$ (D) $\dfrac{\sqrt{2}\sin(2x)}{4\sin\left(x - \frac{\pi}{4}\right)}$ (E) $\dfrac{\sqrt{2}}{4}$

Solution:

$$\frac{1}{\sec x + \csc x} = \frac{\sin x \cos x}{\sin x + \cos x} = \frac{\frac{1}{2} \cdot 2 \cdot \sin x \cos x}{\frac{2}{\sqrt{2}} \cdot (\sin(x)\cos(\frac{\pi}{4}) + \cos(x)\sin(\frac{\pi}{4}))} = \frac{\sqrt{2}\sin(2x)}{4\sin(x + \frac{\pi}{4})}$$

thus we obtain the answer **(C)**.

3. How many distinct real numbers x are there such that

$$\sqrt{x^2+3} - \sqrt{x^2-3} = \frac{3x}{2}?$$

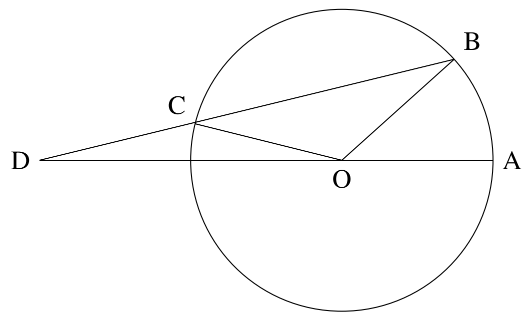
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 or more.

Solution: The domain for the function on the left side is $x \geq \sqrt{3}$ and $x \leq -\sqrt{3}$. Since we know that $\sqrt{x^2+3} - \sqrt{x^2-3}$ is strictly positive, we can focus on only the positive part of x. Since $\sqrt{x^2+3} - \sqrt{x^2-3}$ is a strictly decreasing function and the maximum value of $\sqrt{x^2+3} - \sqrt{x^2-3}$ is $\sqrt{6}$, achieved at $x = \sqrt{3}$, which is less than $\frac{3\sqrt{3}}{2}$, therefore the equation has no solutions, thus we obtain the answer **(A)**.

4. A college math department has 8 different classes that need to be taught and 4 professors available to teach them. If each professor teaches exactly two of the classes, in how many different ways can they be assigned to these classes?
(A) 70 (B) 256 (C) 330 (D) 1680 (E) 2520

Solution: The first professor has $\binom{8}{2} = 28$ ways to choose class, the second professor has $\binom{6}{2} = 15$ ways to choose and the third professor has $\binom{4}{2} = 6$ ways to choose and the last professor has $\binom{2}{2} = 1$ way to choose them. Multiply them together and we get 2520 ways to teach these classes, thus we obtain the answer **(E)**.

5. See the picture below. assume $OB = OC = CD$. Assume $\angle AOB = 42°$. Compute (in degrees) $\angle ADB$.



(A) $14°$ (B) $18°$ (C) $21°$ (D) $24°$ (E) $48°$q

Solution: Let $\angle CDO = x°$, then due to the fact that the exterior angle is equal to the sum of the two remote interior angles, and the base angles of isosceles triangles are equal, we have $\angle COD = x°, \angle OCB = \angle COD + \angle CDO = 2x°, \angle OBC = 2x°, \angle AOB = \angle OBD + \angle ODB = 3x°$. Thus $3x° = 42°$, and we obtain answer choice $\boxed{\textbf{(A) } x = 14°}$.

6. If x satisfies $x^4 + 2x^3 - 22x^3 + 2x + 1 = 0$ and a, b are possible values of $x + \frac{1}{x}$, find $a + b$.
(A) -4 (B) -2 (C) 2 (D) 4 (E) 6

Solution: Dividing the equation by x^2 yields a new equation with the same roots,

$$x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) - 22 = 0,$$

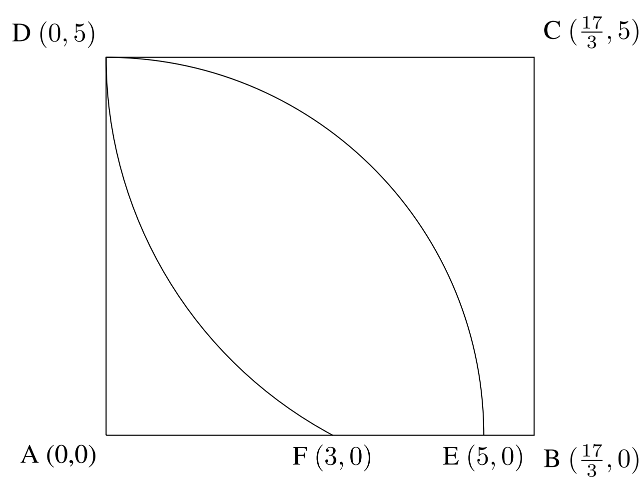
and due to the fact that $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$, we have

$$\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) - 24 = 0.$$

This is a quadratic in $x + \frac{1}{x}$ and it can be solved to yield $x + \frac{1}{x} = -6, 4$ whose sum is $-2 \implies$ **(B)**

7. In Rectangle $ABCD$ with $AB > AD$, a circular arc center at A with radius AD meets $\overline{AB}$ at E and a circular arc centered at C with radius CD meets $\overline{AC}$ at F. If $AD = 5, CD = \frac{17}{3}$, then EF equals :
(A) 2 (B) 3 (C) $\frac{2}{3}$ (D) $\frac{7}{3}$ (E) $\frac{10}{3}$

Solution: We put the rectangle on a coordinate plane with coordinates $A = (0,0); B = (\frac{17}{3}, 0); C = (\frac{17}{3}, 5); D = (\frac{17}{3}, 0)$.
The equation of the circle centered at A has equation $x^2 + y^2 = 25$, so when $y = 0, x = 5$, and therefore the coordinates of point E are $(5, 0)$.
The equation of the circle centered at C has equation $(x - \frac{17}{3})^2 + (y - 5)^2 = 25$, so when $y = 0, x = 3$, and therefore the coordinates of point E are $(3, 0)$.
The distance between these two points is therefore 2, or answer choice **(A)**.



8. What is the largest integer k for which $85!$ is divisible by 42^k?
(A) 3 (B) 13 (C) 41 (D) 81 (E) 135
Note: This question was omitted from scoring on the competition due to the accidental inclusion of the answer on the day of the competition.

Solution: The prime factorization of 42 is $2 \times 3 \times 7$, so we only need to count the number of 7s that appear in $85!$'s prime factorization since 7 is the biggest prime, which can be calculated by $\left\lfloor \frac{85}{7} \right\rfloor + \left\lfloor \frac{85}{7^2} \right\rfloor = 13$, thus we obtain answer choice **(B)**.

9. Let x be the probability that in two flips of a fair coin one is Head and one Tail. Let y be the probability that in four flips of a fair coin two are Heads and two are Tails. Let z be the probability that in five flips of a fair coin there are Heads and two are Tails. which of the following is the correct listing of the order of these numbers?
(A) $x < y < z$ (B) $y < z < x$ (C) $z < x < y$ (D) $z < y < x$ (E) $z < x; x = y$

Solution: Each variable x, y, z is binomially distributed, so they can be easily calculated as follows:

$$x = \frac{\binom{2}{1}}{2^2} = \frac{2}{4} = \frac{1}{2}$$

$$y = \frac{\binom{4}{2}}{2^4} = \frac{6}{16} = \frac{3}{8}$$

$$z = \frac{\binom{5}{3}}{2^5} = \frac{10}{32} = \frac{5}{16}$$

Clearly $z < y < x$, thus we obtain answer choice **(D)**.

10. The number $100!$ is an integer with k digits (in base 10). Then,
(A) $100 \leq k < 150$ (B) $150 \leq k < 200$ (C) $200 \leq k < 250$ (D) $250 \leq k < 300$ (E) $300 \leq k < 5050$

Solution: We use a form of Stirling's approximation: $n \ln n - n < \ln n! < (n+1)\ln n - n + 1$ which can be derived by considering a Riemann sum of $\frac{1}{x}$ with an interval of 1. By multiplying this inequality by $\frac{\log e}{\log 10}$ and applying the change of base formula, we find that

$$n \log n - n \log_{10} e < \log(100!) < (n+1)\log n - n\log_{10} e + \log_{10} e$$

Now, since $e^2 < 3^2 < 10 < e^3$, we know that $\log_{10}(e)$ is between $\frac{1}{2}$ and $\frac{1}{3}$. This gives

$$100 \log 100 - 100 \cdot \frac{1}{2} < \log(100!) < 101 \log 100 - 100 \cdot \frac{1}{3} + \frac{1}{3},$$

or $150 < \log 100! < 169$. Thus, we know that $100!$ has more than 150 digits and less than 170 digits, and thus we obtain answer choice **(B)**.
Note: The actual value of $100!$ *is around* $9.33 \cdot 10^{157}$ *and has 158 digits.*

2 Individual Round II

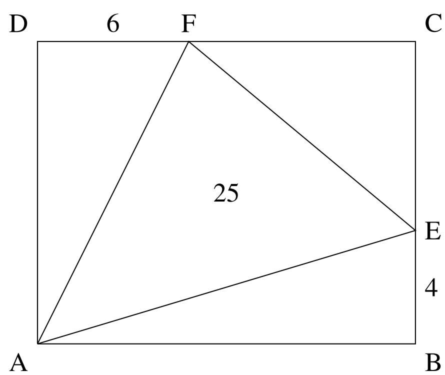
11. Determine all positive real numbers x for which $\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$.

Solution: We utilize the change of base formula as well as properties of logarithms to transform the equation into

$$\frac{\log x}{2\log 2} - \frac{4\log 2}{\log x} = \frac{7}{6} - \frac{3\log 2}{\log x}$$

After substituting $u = \frac{\log x}{\log 2}$ and simplifying, we find the quadratic $3u^2 - 7u - 6 = 0$. This can be solved to yield $u = -\frac{2}{3}; 3$ which yields $x = \boxed{\{2^{-2/3}, 8\}}$

12. Suppose $BE = 4$ cm, $DF = 6$ cm, and the area of $\triangle AEF$ is 25cm^2. Find the area of rectangle $ABCD$.



Solution: Let $AB = x$, and $BC = y$. Then, by considering areas, we have that

$$xy - \frac{1}{2}\cdot 4\cdot x - \frac{1}{2}\cdot 6\cdot y - \frac{1}{2}\cdot(x-6)(y-4) = 25$$

This equation simplifies to $xy = 74$, so we conclude our final answer is $\boxed{74}$.

13. How many positive integers less than or equal to 2018 are divisible by at least 3, 11 or 61?

Solution: First, note that the number of integers less than n and are divisible by k is given by $\left\lfloor \frac{n}{k} \right\rfloor$. Thus, by the principle of inclusion-exclusion, the total number of positive integers satisfying our conditions is

$$\left\lfloor \frac{2018}{3} \right\rfloor + \left\lfloor \frac{2018}{11} \right\rfloor + \left\lfloor \frac{2018}{61} \right\rfloor - \left\lfloor \frac{2018}{3\cdot 11} \right\rfloor - \left\lfloor \frac{2018}{3\cdot 61} \right\rfloor - \left\lfloor \frac{2018}{11\cdot 61} \right\rfloor + \left\lfloor \frac{2018}{3\cdot 11\cdot 61} \right\rfloor = \boxed{814}.$$

14. Form the sequence such that $x_1 = x_2 = 1$, and for $n > 2$, $x_n = x_{n-1}^2 + x_{n-2}$. Of the numbers $x_1, x_2, \cdots, x_{2018}$, how many are divisible by 3?

Solution: We look at the sequence $\mod 3$, and we find that the sequence is periodic with period 8 as follows: $1, 1, 2, 2, 0, 2, 1, 0, 1, 1, \ldots$ Since there are two numbers that have a residue of $0 \mod 3$ in each period of 8, we have that the total number of integers less than or equal to 2016 that are divisible by 3 is $\frac{2016}{8} \cdot 2 = 504$. Now, $x_{2017} \equiv x_{2018} \not\equiv 0 \pmod 3$ so those terms do not satisfy our conditions. Thus our final answer is $\boxed{504}$.

3 Indivdual Round III

15. There are a total of 555 students in a school. For the next term, the number of girls decreases by 17 and the number of boys increases by $\frac{1}{12}$ of the original number of boys. The total number of students decreases by 11. How many girls are in the school in the new term?

Solution: Let the number of girls be g and the number of boys be b and we can obtain 2 equations:

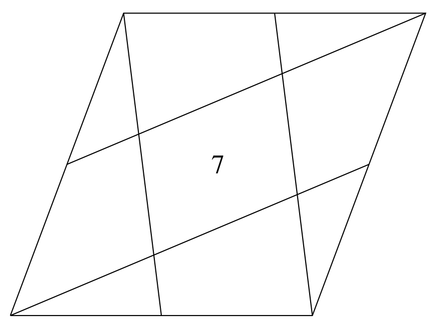
$$\begin{cases} g+b &= 555 \\ g-17+\frac{13}{12}b &= 555-11 \end{cases}$$

Solving for the equation we get that

$$\begin{cases} g &= 483 \\ b &= 72 \end{cases}$$

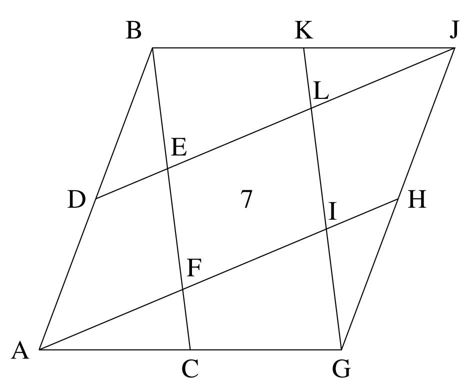
and we get the number of girls in the new term is $g - 17 = \boxed{466}$.

16. Lines from the vertices of a parallelogram to the midpoints of the sides are drawn, as shown, forming a parallelogram of area 7 at the center. What is the area of the original parallelogram?



Solution:

Begin by labeling the diagram,



Since $BD = AD, DE \parallel AF$, we have $BE = EF$. Since $AC = CG, CF \parallel GI$, we have $FC = \frac{1}{2}IG$. Through angle chasing in the parallelograms, we can find $\angle BJE = \angle IAG, \angle EBJ = \angle IGA$, and we have $BJ = GA$, thus $\triangle BEJ \cong \triangle GIA$. Therefore, $CF = \frac{1}{2}IG = \frac{1}{2}EB = \frac{1}{2}FE$, that is, $CF : EF : BE = 1 : 2 : 2$. Since $BKCG$ has the same height as $ELIF$, and $EF = \frac{2}{5}BC$, we have the area of $BKCG$ is $\frac{5}{2}$ the area of $ELIF$. Since the area of $BJGA$ is two times the area of $BKGC$. The area of the original parallelogram is $5 \times 7 = \boxed{35}$.

17. Count the number of integers so that $1 \leq n \leq 1000000$, and the sum of all digits in n is 11.

Solution: Clearly, 1000000 does not meet the criteria, so we look at integers from 1 to 999999. If we let a_i be the i th digit of the number ($a_1 = 0$ for 5 digit numbers, etc), then the conditions reduce to: $\sum_{i=1}^{6} a_i = 11$, and $0 \leq a_i \leq 9$. This is a well known stars-and-bars problem, and the number of solutions without considering the upper bound of a_i is $\binom{16}{5} = 4368$.
Now in order to take into account the conditions, we must subtract the solutions gotten with at least one of $a_i > 9$. We have two cases to consider:

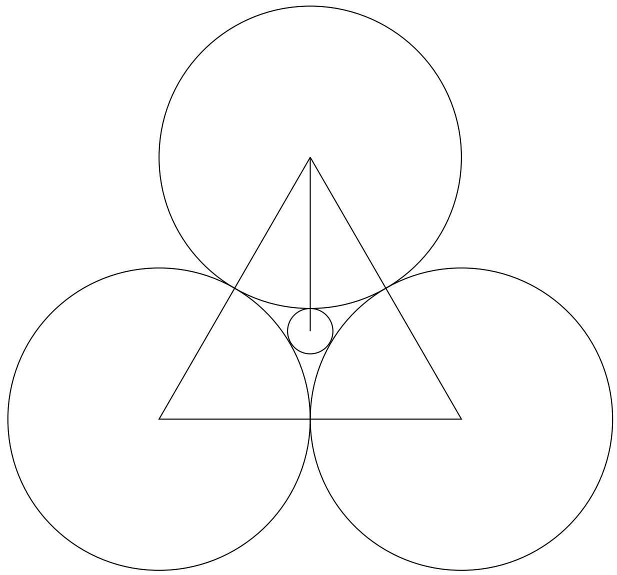
Case 1: $a_i = 11$. This means that the rest of the digits are 0. We have 6 choices for a_i, so overall we overcounted by 6 in this case.

Case 2: $a_i = 10, a_j = 1$. This means that the rest of the digits are 0. We have $6 \cdot 5$ choices for the pairs (i, j) so overall we overcounted by 30 in this case.

Thus our final answer is $\binom{16}{5} - 30 - 6 = \boxed{4332}$.

18. Three congruent right circular solid cones, each of volume equal to 9, are placed in space so that their bases lies on a given horizontal place with each one's base touching the other two and these cones all point upward. A fourth solid cone, congruent to the first three, is turned point down, placed into the space between the first three, and allowed to fall as low as possible. The volume of that portion of the fourth cone that lies below the place is a number than can be expressed in the form $a\sqrt{b} - c$, where a and c are positive integers and b is a positive square free integer. Computer $a + b + c$.

Solution: Let us look at the planar cross section of the cones:



Let r be the radius of the outer cones and R be the radius of the inverted cone on the plane. The centers of the outer cones form an equilateral triangle with a radius $2r$. Since equilateral triangles' center is the same as the centroid, we must have that the length from one corner to the center is $\frac{2}{3}r\sqrt{3}$. But geometrically, that distance is $r + R$. So, we have $r + R = \frac{2}{3}r\sqrt{3}$ or $R = \left(\frac{2}{3}\sqrt{3} - 1\right)r$. Since the volume of cones is proportional to r^2h and the height is proportional to the radius, we must have that the volume below the plane of the inverted cone is $\left(\frac{R}{r}\right)^3 V_{original}$, which simplifies to $26\sqrt{3} - 45$. The sum of these numbers is what we seek, resulting in our answer $\boxed{74}$.

4 Team Round

1. Four people are in a room. For each possible grouping of three of them, compute the sum of the average of the three plus the age of the fourth. The numbers we get this way are 29,23,21, and 17. Find the difference between age of the oldest of them and the age of youngest of them.

Solution: Let a, b, c, d be the ages of the people. Then, we obtain the following equations:

$$\begin{cases} \frac{a+b+c}{3} + d &= 17 \\ \frac{a+b+d}{3} + c &= 21 \\ \frac{a+c+d}{3} + b &= 23 \\ \frac{b+c+d}{3} + a &= 29 \end{cases}$$

By summing up all the equations, we find that $2(a+b+c+d) = 17+21+23+29 \implies a+b+c+d = 45$. Substituting this into the fractions in the above equations gives:

$$\begin{cases} \frac{45-d}{3} + d &= 17 \\ \frac{45-c}{3} + c &= 21 \\ \frac{45-b}{3} + b &= 23 \\ \frac{45-a}{3} + a &= 29 \end{cases}$$

Which can be easily solved to give $(a, b, c, d) = (21, 12, 9, 3)$ so the difference we are seeking is $21 - 3 = \boxed{18}$.

2. Find all positive values for b for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges.

Solution: By the test for divergence, we have that $b < 1$, and thus the series is decreasing.

As the series is finite when $n = 1$, we can ignore that value in the determination of convergence and divergence. Due to the fact that the series is decreasing, we have that $\sum_{n\geq 2} b^{\ln(n)} \leq \int_1^{\infty} b^{\ln(n)}\, dn$ This integral can be evaluated through the use of a u-substitution $u = \ln(n)$ which results in the integral $\int_0^{\infty} (be)^u\, du$ which only converges when $b < \frac{1}{e}$, and thus the series converges on the same interval.

To prove that the values $b \geq \frac{1}{e}$ do not converge, we can instead estimate the summation in the following form: $\sum_{n\geq 2} b^{\ln(n)} \geq \int_1^{\infty} b^{\ln(n+1)}$ which can likewise be evaluated with the u-substitution $u = \ln(n+1)$ to obtain that the sum is less than the integral $\int_{\ln(2)}^{\infty} (be)^u\, du$ which diverges when $b \geq \frac{1}{e}$.

Thus, the series converges when $\boxed{b < \frac{1}{e}}$ (for positive values of b)

3. If $\sin x + \cos x = \frac{1}{3}$, what is the value of $\sin^3 x + \cos^3 x$?

Solution: Squaring the equation we can get that $\sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{1}{9} \rightarrow \sin x \cos x = -\frac{4}{9}$.
We know that $\sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x) = \dfrac{1}{3} \cdot \dfrac{13}{9} = \boxed{\dfrac{13}{27}}$

4. Consider a 3×3 array of squares. Color each square one of three possible colors (red, yellow, or blue) so that no two squares that share and edge have the same color. How many ways are there to do this coloring?

Solution: Without the loss of generality, suppose the center has color A, then the edges can only have color B or C, then we can divide them into cases:

1.

	B	
B	A	B
	B	

the colors for all the edges are the same, then for each corner we have 2 ways to choose its coloring, and we have 2 ways to color the edges (all B or all C), thus we have $2^5 = 32$ ways.

2.

	B	
B	A	B
	C	

We can have 3 of the edges to be one color and 1 to be another, then we have 2 ways to color upper 2 corners and 1 ways to color the bottom 2 corners, and we have 2 ways to choose to color them, and 4 results due to rotation of this figure, thus $2^2 \cdot 4 \cdot 2 = 32$ ways.

3.

	B	
B	A	C
	C	

We have 2 ways to color the upper left and the lower right corner, and 4 ways to rotate the squares, thus $2^2 \cdot 4 = 16$ ways

4.

	C	
B	A	B
	C	

There is only 1 way to color this case and we can rotate this figure 2 ways.

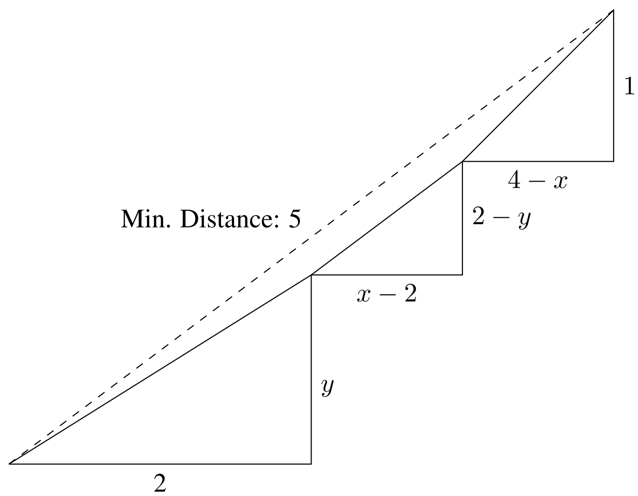
Since A can be any of the three colors, we have $(32 + 32 + 16 + 2) \cdot 3 = \boxed{246}$ total ways to color this figure.

5. Find the minimum value of $f(x, y)$ over all possible real numbers x and y:

$$f(x, y) = \sqrt{4 + y^2} + \sqrt{(x-2)^2 + (2-y)^2} + \sqrt{(4-x)^2 + 1}$$

Solution: We interpret the problem geometrically. The three square roots are merely the hypotenuses of right triangles with legs $(2, y), (x-2, 2-y)$, and $(4-x, 1)$. Suppose that all three triangles are stacked (see diagram below). Since the total horizontal and vertical lengths are constant, the problem corresponds to finding the minimum distance between the bottom left point and the top right point. But this is no more than $\sqrt{(2+x-2+4-x)^2+(y+2-y+1)^2} = \sqrt{3^2+4^2} = \boxed{5}$.

Note: The actual (x, y) at which the minima occurs at is $(x, y) = (\frac{8}{3}, \frac{3}{2})$



6. In triangle ABC, assume $AB = AC$ and $AC < BC$. Extend $\overline{CA}$ to E so that $BC = CE$. Then extend $\overline{AB}$ to point D such that $AD = DE$. It turns out that we also have $AD = BC$. Compute (in degrees) $\angle ACB$.

Solution:
Let $AC = AB = b$, $BC = CE = DE = AD = a$. Then $EA = a - b$, $BD = a - b$.
Let $\angle ACB = x^\circ$, then $\angle ABC = x^\circ, \angle DEA = 2x^\circ, \angle AED = 2x^\circ$.
Thus, in $\triangle ADE$,

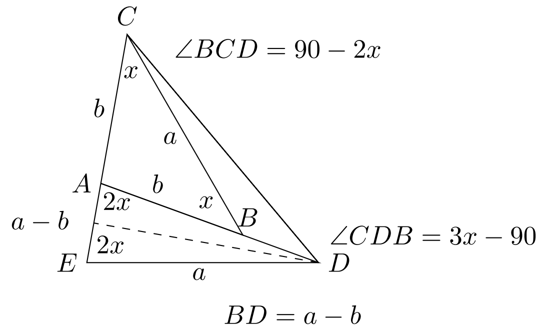
$$\cos(2x) = \sin(90 - 2x) = \frac{\frac{a-b}{2}}{a} = \frac{a-b}{2a} \tag{1}$$

Since $EC = ED$, $\angle EDC = \angle ECD = \frac{180 - \angle CED}{2} = 90 - x, \angle BCD = \angle ECD - \angle ECB = 90 - 2x$
$\angle ADE = 180 - 4x, \angle CDB = (90 - x) - (180 - 4x) = 3x - 90$.

Thus in $\triangle BCD$, using law of sines,

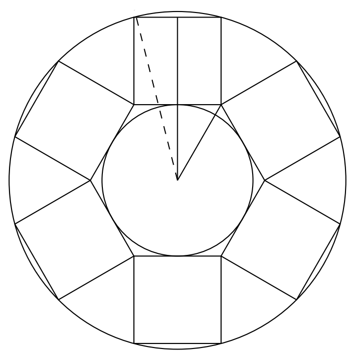
$$\frac{\sin(90 - 2x)}{a - b} = \frac{\sin(3x - 90)}{a} \tag{2}$$

Combining equation (1) and (2), we get $\sin(3x - 90) = 1/2$. Thus $x = 40^\circ$ or $x = 70^\circ$. Since $BC > AC, x = 70^\circ$ does not work.
Therefore, $\boxed{x = 40^\circ}$ is the solution.



7. Six congruent squares are externally tangent to Circle C_1, arranged in a non-overlapping fashion so that each square shares one vertex each with two other squares. Circle C_2 (which shares a center with C_1) is circumscribed about all of the squares. Compute the fraction (area of C_2)/(area of C_1).

Solution:



Suppose that the center circle has radius r. Then, the hexagon around the circle, as well as the squares, has side length $s = 2r\tan(30^\circ) = \frac{2}{\sqrt{3}}r$. Then, the radius of the larger circle is $R = \sqrt{\left(\frac{s}{2}\right)^2 + (r+s)^2}$ by the Pythagorean theorem. The ratio of the areas is the square of the ratio of the side lengths, so we obtain

$$\left(\frac{R}{r}\right)^2 = \frac{\left(\frac{s}{2}\right)^2 + (r+s)^2}{r^2} = \boxed{\frac{8+4\sqrt{3}}{3}}$$

8. Define a sequence by $a_0 = 0, a_1 = 1$, and for $n \geq 2, 6a_n = 11a_{n-1} - 5a_{n-2} + 11$. What integer is closest to a_{1000}?

Solution: In order to solve this question, we utilize generating functions. By multiplying both sides of the recurrence by x^n and summing each side, we find that

$$6\sum_{n\geq 0} a_n x^n = 11\sum_{n\geq 0} a_{n-1}x^n - 5\sum_{n\geq 0} a_{n-2}x^n + 11\sum_{n\geq 0} x^n$$

After shifting indices and factoring out powers of x, we find that

$$6\sum_{n\geq 0} a_n x^n = 11x\sum_{n\geq -1} a_n x^n - 5x^2\sum_{n\geq -2} a_n x^n + 11\sum_{n\geq 0} x^n \tag{3}$$

$$= 11x\left(\frac{a_{-1}}{x} + \sum_{n\geq 0} a_n x^n\right) - 5x^2\left(\frac{a_{-2}}{x^2} + \frac{a_{-1}}{x} + \sum_{n\geq 0} a_n x^n\right) + 11\sum_{n\geq 0} x^n \tag{4}$$

Note that through the recurrence, we find $a_{-1} = 1$ and $a_{-2} = \frac{22}{5}$. We let $f(x) = \sum_{n\geq 0} a_n x^n$, and after using the standard geometric summation and simplifying, we find

$$f(x) = \frac{-11 - 5x + \frac{11}{1-x}}{6 - 11x + 5x^2}$$

After using partial fraction decomposition on f, we find

$$f(x) = \frac{11}{(1-x)^2} + \frac{60}{1-5x/6} - \frac{71}{1-x}$$

so after expanding out all the series, we find that the coefficient of x^n is $a_n = 11(n+1) - 71 + 60(\frac{5}{6})^n$. So, the nearest integer to a_{1000} is $11 \cdot 1001 - 71 + (60(\frac{5}{6})^{1000} \approx 0) = \boxed{10940}$.

Typeset in LaTeX
Last updated March 4, 2018