

Math Day at the Beach 2008

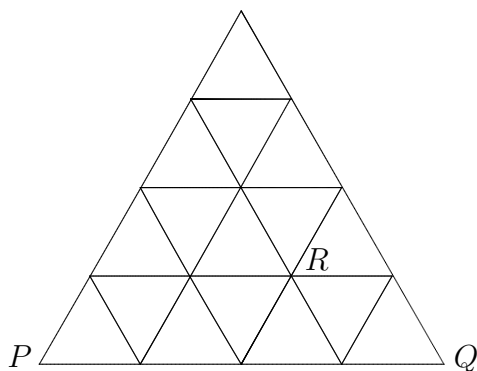
TEAM ROUND – Write your name and school and mark your answers on the answer sheet. No calculator is allowed. You have 30 minutes to work on these problems.

1. Let a be a positive number such that $a^{2009} - 2a + 1 = 0$. Find all possible values of the sum $S = 1 + a + a^2 + \cdots + a^{2008}$.
2. Let $f(x, y) = (1 + \sqrt{x} + \sqrt{y})(1 + \sqrt{x} - \sqrt{y})(1 - \sqrt{x} + \sqrt{y})(1 - \sqrt{x} - \sqrt{y})$. Compute $f(7, 17)$. (It will be an integer.)
3. One leg of a right triangle has length 20 and the radius of the inscribed circle is 6. Compute the length of the hypotenuse of the right triangle.
4. Write down the digits between 1 and 9999 as one concatenated string, i.e.

123456789101112....

What is the 10001st digit in this string?

5. A line intersects the curve $y = x^3$ at the point (a, a^3) for some $a > 0$, and at exactly one other point where the line and the curve are tangent. Compute the area of the region enclosed by the line and the curve. Express the answer in terms of a .
6. A hemisphere of radius 2 is lying on the ground with its flat side down. Inscribe an equilateral triangle into the circle that is the base of this hemisphere. Compute the surface area of that portion of the hemisphere which lies directly above the triangle.
7. How many different paths are there from P to Q but not passing through R along edges of this figure if each step must move further to the right?



8. The integers a , b , and c are each greater than 20. One of them has an odd number of positive divisors; the other two each have exactly 3 positive divisors. If $a + b = c$, compute the smallest value for c .