Math Day at the Beach 2008

INDIVIDUAL FREE RESPONSE – *Write your name and school and mark your answers on the answer sheet. You have 25 minutes to work on these problems. No calculator is allowed.*

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16. When $N!$ is written in base 8, it ends in exactly 85 consecutive zeros. Find the largest N for which this is true. Express your answer in base 10.

17. Suppose $A = (x, 5)$, $B = (-2, y)$ and $C = (1, 1)$ are colinear, and $AC = 2 \cdot BC$. Find all the possible pairs (x, y).

18. A right circular cone has a base with radius 2 and altitude $2\sqrt{3}$. A point X is chosen at random inside the cone. What is the probablity that this point is closer to the base than to the side of the cone?

19. Two 10 inch by 12 inch pieces of cardboard are attached along their longer edges to make a hinged folder. A string of length w connects the lower corner of the front cover to the upper corner of the back cover. This string allows the folder to open until the angle between the two covers is θ such that $\sin\left(\frac{\theta}{2}\right) = \frac{1}{4}$. Find w.

20. The numbers $x_1, x_2, x_3, \ldots, x_8$ are all either 1 or -1. Let $S_1 = x_1, S_2 = x_1 + x_2$, and in general $S_n = x_1 + x_2 + \cdots + x_n$. How many different ways are there to choose these numbers so that $S_8 = 0$ and the maximum value of S_n is 3?